A new enabling technology for learning and teaching quantitative skills

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Executive Summary

The ability to understand and apply mathematical principles is essential to building and sustaining knowledge and expertise in the sciences, statistics, economics,, engineering, information technology and many other areas. It is widely recognised that students at all levels of education are increasingly struggling with learning mathematical material, and disengaging from the discipline area. This has been highlighted by successive federal governments as an area of substantial concern to Australia, and the problem is repeated overseas.

The broad aim of this project was to develop tools and approaches to enhance the quantitative and mathematical skills of students at the lower tertiary and upper secondary levels. The key approach was creation of a flexible electronic framework through which students and educators can interactively generate a very large number of illustrative examples, problems and questions that cover a wide range of fundamental mathematical, statistical and quantitative skills.

The specific primary project goal was to produce a much more sophisticated resource than extant electronic mathematics teaching tools, which only generate simple randomised variants of well-defined questions along with the corresponding simple numeric answers.

In contrast, the project envisaged a system that would generate a suite of random questions and corresponding fully worked, formatted solutions to every question, clearly and unambiguously reproducing the steps that students would typically take when solving that problem. This would provide students and educators with a mechanism for concentrating on those concepts which cause them difficulties, seeing every step taken to solve such problems and thus improving their technical and creative problem-solving abilities. Importantly, the randomisation within the system must be much more than simply “changing numbers”, instead including substantial variation in the core content of each example.

The primary aims of this project have been achieved, with a powerful and flexible system having been implemented. There are now 138 question templates, each of which provide access to questions, short answers and fully worked solutions to one or more questions of a specific type. This content covers a large component of the essential knowledge from advanced secondary mathematics and introductory tertiary mathematics. It also includes content from science, engineering and business.

The system has been deliberately designed to be of direct use to students studying a wide range of discipline areas. Key mathematical concepts tend to be ubiquitous, so the materials can be used within Australia and internationally. The software comprising the system is flexible and modular, so future third-party contributions to the system will be encouraged, with an appropriate system of quality control.

During the course of the project, a number of extensions were identified (such as expanding the content to include new discipline areas, supporting computing questions and developing a more structured user interface). As a result, additional funding support has been leveraged and development will continue until at least the end of 2008.

An initial announcement of the system was made at the 11th Annual Conference on Education and Virtuality in 2007, and further announcements will be made during 2008 and 2009 at conferences and on websites after a comprehensive web-based interface that is currently under development goes live.

Table of contents.

In addition, an attached document shows sample questions, short answers and fully worked solutions generated by the system, illustrating the particular question templates that are currently supported.

1. Aims

Mathematics is essential to building and sustaining knowledge and expertise in the sciences, statistics, economics, physics, engineering, information technology and many other discipline areas. Unfortunately, it is becoming increasingly clear that students at the tertiary and secondary levels are increasingly struggling with learning mathematics, and disengaging from the discipline area.

The broad aim of this project was to develop tools and approaches to enhance the quantitative and mathematical skills of students at the lower tertiary level (and hence also at the upper-secondary level). The key approach was the creation of a flexible electronic framework through which students and educators can interactively generate a very large number of illustrative examples, problems and questions that cover a wide range of fundamental mathematical, statistical and quantitative skills.

The main goals and features of the software include:

- automatically generating a suite of random questions and corresponding fully worked, formatted solutions to every question, clearly and unambiguously reproducing the steps that students would typically take when solving the problem;
- providing students with a mechanism for concentrating on those concepts which cause them difficulties, enabling inquiry-based learning and improving their technical and creative abilities;
- implementing a powerful learning aid that gives support for both introductory and advanced mathematical concepts and processes;
- making use of interactive web-based presentation of solutions;
- allowing instructors to efficiently and easily create resources for illustrative examples, practice materials and individualised assessment;
- being directly usable in all discipline areas that require quantitative skills; and
- free availability to the education community as open-source software, with a modular design allowing components to be easily redesigned and extended.

2. Intended outcomes

The key intended outcome for this project was development of a flexible, electronic system that fulfilled the goals specified in the Aims, and to make this system available to:

- students, who use the system to learn mathematical and quantitative concepts
- teachers and educators, who can use the system as a flexible teaching aid
- potential contributors, including teachers, students, programmers and others, who can design and add new material and features to the system.

Specific features of this system, as described in the initial application, are for it to:

- implement a range of fundamental mathematical operations (such as manipulating fractions, solving equations, solving linear, simultaneous and quadratic equations, performing algebraic manipulations, functions, differentiation, set-theory and logic, matrices, and methods of proof).
- in addition to calculating the answer, the system will also produce fully formatted output of each stage in the process, with each step corresponding to the working that a student would be likely to apply at that stage.
- assist students in developing their higher-level skills, by giving them access to an effectively unlimited collection of practice materials that illustrate both the large-scale and more subtle similarities and differences between mathematical concepts. The variation must be much more sophisticated than merely using random numbers, but must also use randomness in the structure of the questions, including algebraic and mathematical content.
- be very easy to use by instructors. Slight (or even substantial) changes to questions should be handled at the user-level, rather than at the software-level. Thus, the system must include an interactive front-end, which asks the instructor specific questions designed to determine exactly what should be included in the problem and what the restrictions are on parameters within the problem. These choices will only need to be selected at the time the structure of the question sheet is being initially created. Each time the actual randomised questions are generated, no input is required from the instructor.
- provide support for parsing and interpreting material specified by the instructor in a simple meta-language, which provides an easy front-end to the underlying software routines.
- easily be extended and broadened, without requiring extensive software redesign. Thus the system should support continued evolution and development over time, with contributions from non-experts.
- include a substantial breadth, depth and variety of mathematical content. The system should ultimately incorporate materials for all, or most, first-year tertiary-level mathematics and statistics courses.

- benefit a broad range of users and disciplines. The system should be equally useful in the many and varied fields that apply mathematics to the solution of practical problems. Fields of particular interest include business, economics, engineering and biology. In addition, the system should be suitable for use at the tertiary level and also for advanced secondary studies.
- include a web-based interactive component, allowing users to access partial solutions and key steps.
- based on standard software tools and techniques, to ensure maximum uptake and dissemination. All questions and solutions must be written in Latex code, and all electronic presentation done using PDF.

The actual outcomes of the project are described in Section 4.

3. Approach and methodology

The primary goal of this project is to develop a flexible, useful software system that aids in learning and teaching core mathematical and quantitative skills for students making the transition to tertiary education. The particular technical skills that should be covered are well-defined, so the main focus of the approach and methodology was the efficient development of an effective and flexible system that provides access to useful and correct learning aids. Here we first discuss the driving principles for designing the system, then describe the structure of the resulting system from the perspective of a user or contributor, then briefly discuss some technical details of the system architecture.

3.1 Driving principles

Mathematics is essential to building and sustaining knowledge and expertise in the sciences, statistics, economics, physics, engineering, information technology and many other discipline areas. It is well-recognised that there are major problems associated with secondary and tertiary mathematics education, and many students are increasingly struggling with understanding and applying mathematical and quantitative concepts. Universities all around the world are finding that these problems are further exacerbated by such factors as increasing class sizes, a greater breadth and diversity of student educational backgrounds and experiences and differences in students' levels of motivation and their professional aspirations. At the same time, there are increasing demands on academic staff time, which can lead to a decrease in students' perceived quality of their educational experience.

At the same time, some teachers of secondary-level mathematics may have limited mathematical backgrounds, with few opportunities to develop effective strategies for responding to students' learning difficulties and the diverse expectations.

There are numerous research projects, conferences and publications devoted to addressing the problems facing mathematics education, and there are many factors that can help students to achieve success. These factors include being taught by enthusiastic and talented instructors, appropriate use of technology, and access to interesting and relevant teaching and learning activities. The development of a variety of high-quality, time-efficient, mathematically sound educational resources is an important way to support students and empower mathematics educators to respond to these challenges.

It is widely accepted that success at mathematics requires a combination of skills: technical knowledge is important, but of equal or greater importance is the ability to use intuition, flair and elegance when solving problems. Lectures are an important part of the learning process, as they provide students with the opportunity to observe an experienced practitioner applying these creative talents, and explaining and demonstrating how to do so. However, a common thread to mathematics learning experiences is that material cannot be absorbed and assimilated passively. It is learned by **doing**, not simply by **watching**. At every level (up to and beyond PhD), inquiry- and discovery-based learning are essential, and students must work through

many examples and problems in order to hone their technical and intuitive skills. By thinking about what they are doing and observing the similarities and differences between various questions they become attuned to patterns and subtleties, thus improving their ability to choose what techniques to use and how to work creatively.

Traditionally, working through examples was sometimes regarded merely as rote learning. Certainly there is some need to commit mathematical facts to memory, but we are not suggesting that the primary reason for working through a number of questions is simply to learn how to recite facts. Instead, learning mathematics by practicing is a genuine and necessary aid to improving understanding and enhancing creative abilities, in addition to learning technical skills.
These general principles informed the Approach and Methodology used to develop the software package that forms the primary outcome of this project. This software automatically generates a suite of random questions and corresponding fully worked, formatted solutions that illustrate a range of important mathematical and quantitative concepts, clearly and unambiguously reproducing the steps that students would typically take when solving such problems.

Randomization in computer-based learning resources is not new. However, successfully using it in effective aids for learning mathematics has previously proved to be problematic. Some well-known and excellent mathematics packages simply give the final answer, with no indication of any intermediate steps or processes that are required in order to actually derive the answer. If a student makes a mistake, the only option is to go back and try again, often leading to repetition of the error and a loss of confidence.

The software developed in this project overcomes some of the limitations of existing electronic and book-based methods of providing examples and questions as aids to teaching and learning mathematics.

3.2 System Description

The developed software implements a system that generates sets of questions and corresponding fully worked solutions that cover a range of mathematical concepts. Questions include significant random variation, not just in numerical constants but also in their algebraic content. This can be used to create an unlimited set of extra practice questions for students and teachers, and can also be used to create personalised assessment items.

From the user's perspective, there are three levels of abstraction in the system: the software itself written in Java modules (the lowest level); Problem templates; and Assignment files. This hierarchy is illustrated in Figure 1. Assignment and template files may be divided into sections that define a set of output files. The sections identified in Figure 2 are Questions, (fully worked) Solutions, and (short) Answers.

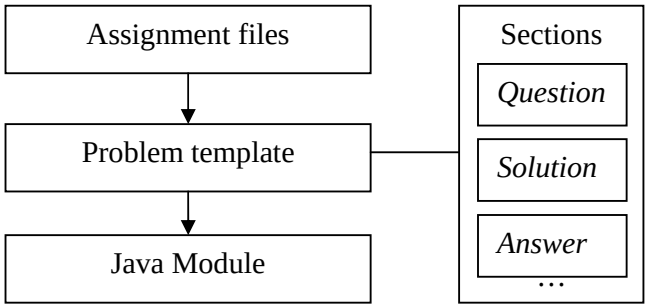


Figure 1: The three levels of abstraction in the system.

The software is based on Java modules, each of which corresponds to the particular quantitative problem, or class of problems. Each time a new mathematical concept is introduced into the system, Java code is written to implement the mathematical steps taken in solving problems that illustrate this concept. This requires a reasonable level of programming expertise, but once developed, these modules can be used in a variety of theoretical and practical questions.

Sitting one level above the Java modules are the Problem templates. These normally contain LaTex code (but other formats are also possible), and may include calls to relevant Java modules. Templates currently have three sections, corresponding to questions, short answers and fully worked solutions. For each mathematical question the system produces three output files, one containing the formatted question, one giving just the final answer to the question, and one containing the formatted fully worked solution. However, users are able to create their own sections, depending on their specific needs. Another useful feature incorporated into problem templates is the ability to pass parameter strings into the Java modules. Problem templates are usually created by a learning designer, and require knowledge of LaTex formatting but not Java programming.

One level above the problem templates are the Assignment files. These files combine three simple control operators (CALL, REPEAT, and MULTI) with the names of Problem templates. This allows the user to create various assignments. This gives great flexibility as to the type and number of questions that will appear in each assignment. Of course, each question will be generated using randomness, and the fully worked solutions will accurately reflect the working required to solve each specific question. Creating assignments can be easily performed by anyone with basic computer experience. Therefore, teachers and students have a simple and easy tool, which is sufficiently flexible and powerful to compile quickly sets of problems, assignments, and examples for self-study, class work or assessment.

3.3 Technical System Architecture

In this section we present some technical information about the architecture, structure and design of the software package. The key components of the system link together as shown in Figure 2, and a brief description of each component is given below.

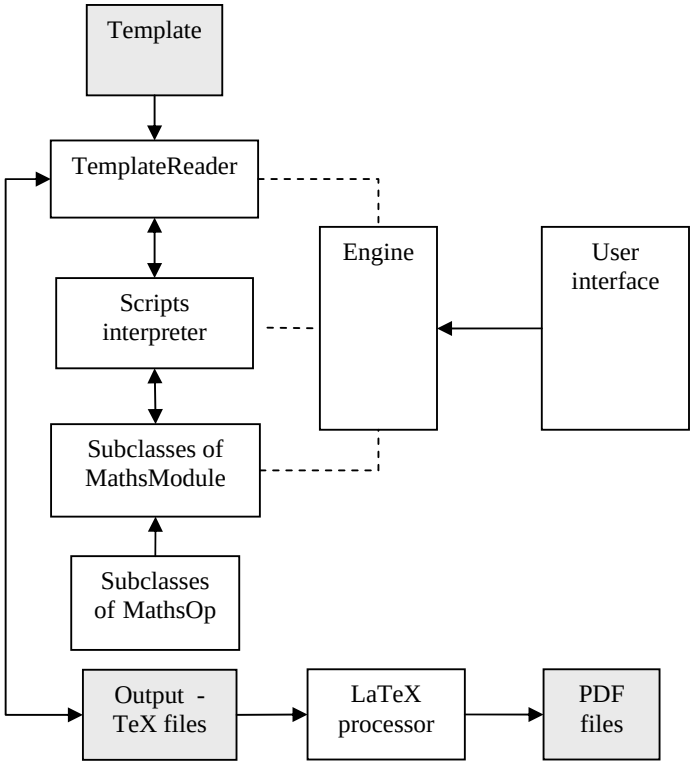


Figure 2: System architecture and information flows.

3.3.1 Template

Each template is a text file that includes formatting and some special keywords. Templates are read by the templates processor, which has the following capabilities:

- Template nesting: other templates can be called using the CALL operator. This causes the system to include the result of processing nested templates into the text of the current template.
- Randomisation of text: the operator MULTI allows the system to randomly select one or many blocks of text from the specified set of blocks inside the template. For instance, the user may want to create an assignment with two randomly chosen questions out of ten available templates on a particular topic. To achieve this, the user will list all ten templates (using the operator CALL) in the template, and then apply MULTI 2 to randomly select two.

- Cycles: the operator REPEAT causes the system to repeat the chosen part of the template a specified number of times.
- Sectioning: currently, all output is separated into three sections, comprising questions, short-answers and fully worked solutions. The user may change this, and each section of the template will be processed and written in a corresponding file.
- Script calls: script calls are passed to the script interpreter, described below.
- TeX/LaTeX commands: any TeX code within a template is passed directly into the output files without any further processing by the system. The standard LaTeX system is used to process these formatting commands after the output files are generated.
- Reading parameters from external data files. For example, if the user requires the module to generate a specific question using real data (that is, data from a file or table that represents some real phenomenon rather than randomly generated data), this can be specified in the template using the command <DATA_STORAGE_TYPE> <DataSourceName>("path"), and the system will read a line of data (a random line or chosen according to some user-specified criteria) from the specified file.

3.3.2 Script interpreter

Script calls within templates are processed by the script interpreter, which does the following:

- creates MathsModule class objects, optionally passes a string of parameters into the module, and runs the module; and
- reads the output text block from the current module and inserts it into the template text in order to create the new output files.

These capabilities of the script interpreter, together with control structures within the templates, are sufficient for easy creation of questions and solutions. In future, a more comprehensive Script interpreter could be developed.

3.3.3 Modules

The MathsModule subclasses contain Java code to generate questions and solutions.

3.3.4 Engine

The Engine class is the core of the system, enabling information flows between all components of the system.

3.3.5 User interface

The system has a simple command line interface, but this could be replaced by a more sophisticated multifunction GUI.

3.3.6 MathsOp subclasses hierarchy

Typical mathematical expressions and equations, even quite complex ones, are often based on a comparatively small number of mathematical operations (such as arithmetic, algebraic manipulation, logic and simple calculus). The system is based around a tree of java classes, responsible for these operations.

3.3.7 Open architecture, extensibility and platform independency

One of the key goals of the system is to ensure a maximum level of openness and extensibility. Keeping this in mind, we chose an open source development model with Java as a main programming language. The resulting high level of system flexibility is reflected in the following features:

- Templates and modules of the problems are easily edited, added or swapped, and the set of modules is identified dynamically at runtime.
- The template processor, script processor and user interface can potentially be replaced or upgraded without redesigning any other components. Possible candidates include replacing the command line interface with a GUI, or replacing the simple script interpreter with the python scripts processor.
- Adoption of a GPL licence allows any interested parties to participate in the ongoing development process.
- Using multi-platform Java makes the system platform independent, so it can run on practically any commonly used operating system.

4. Actual outcomes: advances in knowledge

Rather than investigating a particular problem or conducting a review of approaches to a particular issue, the goal of this project was production of a novel software package to support learning and teaching mathematical and quantitative skills. As identified in the application, successful development of such a system would advance the ALTC's funding priorities *Innovation in teaching and learning, particularly in relation to the role of new technologies and Strategic approaches to learning and teaching that address the increasing diversity of the student body*.

The primary aims of this project have been achieved, with a powerful and flexible system having been implemented. There are three key groups for whom the system is designed: students, educators and future contributors. Hence the discussion of outcomes includes material primarily directed to each of these groups.

Details of the system as it is on completion of the ALTC-funded project are given in the following sections. Section 4.1 describes the key application programs that comprise the system, Section 4.2 includes a list of all content modules and question templates that are currently available, and Section 4.3 describes examples of output arising from these content modules. Thus, Section 4.1 is of particular importance to future contributors, Section 4.2 to educators, and Section 4.3 to students and educators.

Note that development of the system is continuing with support in addition to the original ALTC funding, so in some places the system components are described as being in testing, or in active development.

4.1 Key application programs.

The developed system includes the following six key applications that can be used by students, educators and future contributors of materials.

4.1.1 **Command Line Application Module** (in operation).

A deliberate feature of the system design is that the user interface is completely separated from the generation of questions and assignments. Therefore it is possible to create several user interfaces of different types (such as command line, GUI, web-based) according to specific user needs and requirements. The Application module is a simple command-line interface that supports basic functionality, including generation of questions, answers and solutions, and compiling assignments based on the given assignment template. The command line parameter for this module is the template name, and the output comprises the generated questions, short answers and fully worked solutions. Despite its simplicity, using this interface is preferable in many cases, such as processing multiple assignments at once (in a package), or working in an environment where a GUI is not accessible (for instance, on web-server).

4.1.2 **Edit Settings Module** (in operation).

This module provides a GUI for entering and tuning system parameters and settings, such as paths to modules and templates. It is primarily intended for usage with the Application Module described above.

4.1.3 **Interactive Application Module** (in final testing).

This module provides an interactive GUI-based interface for the system. The functionality provided by this interface is significantly extended compared to the command line application module. In addition to interactive generation of learning materials, the interactive application module includes support for:

- browsing and editing a tree of templates, in which each node of the tree corresponds to a specific operator within the template
- browsing and editing a tree of generated assignments (questions, short answers and fully worked solutions)
- generation and presentation of PDF/DVI files
- capability to regenerate a single node in the assignment tree without the need to regenerate the entire tree
- interactive editing of assignment latex-code
- saving results in tex, pdf or dvi files
- editing settings (similar to the Edit Settings Module).

4.1.4 **Template Repository Application Module** (in testing).

This module implements a template repository and GUI for saving templates and auxiliary files (such as graphics files and additional latex packages) required for template processing. The repository also maintains metadata about question templates (including a short description of the question, authors, key words and update history) and samples of generated questions in pdf-files. This template is based on a SQL database.

4.1.5 **Assignment Constructor Module** (in testing).

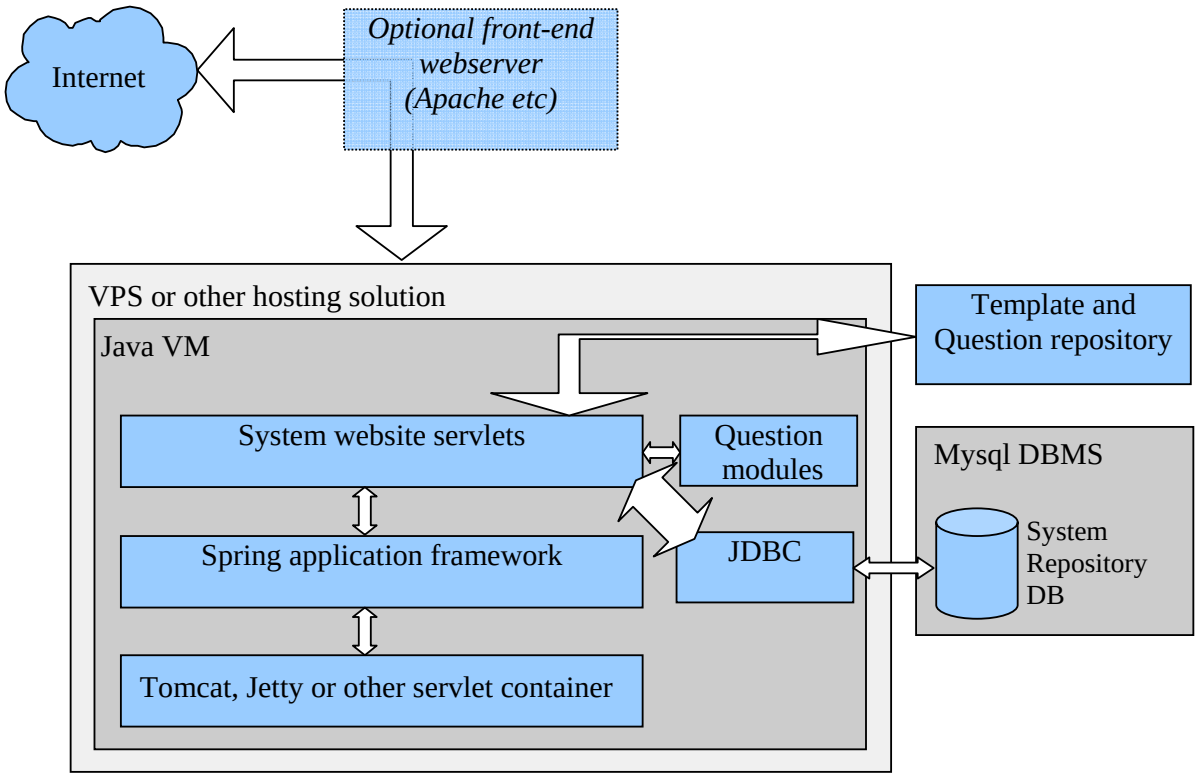
This module is designed to provide easy interactive composition of assignments from question templates stored in the Template Repository, and is intended for users who are completely unfamiliar with latex editing. The module allows rapid composition of the sets of available questions and generation of assignments (and also supports a range of control operators that provide support for conditional and loop-based processing). After construction, assignments are then processed by the Command Line Application Module or the Interactive Application Module.

4.1.5 **Web Interface Module** (in development).

To facilitate easy access by students, educators and contributors, all components of the system are being organised into a robust and systematic single entity, with a web-based front-end access. This represents a substantial extension and improvement to the initial specification. Advantages of employing a front-end web server include:

- integration of system content into existing WWW portals
- better security
- better management of static content
- provision of performance tuning, including caching and load balancing.

A schematic diagram of the overall system is shown.



4.2 Content modules and question templates.

Each of the mathematical and quantitative concepts embedded in the system has been modularised into a separate template. Each template provides support for one or more types of question based closely on the particular conceptual topic being illustrated. When designing a learning activity, the user can select which template(s) they wish to incorporate in that activity. Each template generates questions, short

answers and corresponding fully worked solutions. Most templates generate one particular type of question, but some templates generate different types of question depending on the specific input parameters.

Sections 4.2.1 to 4.2.12 briefly describe all of the 138 templates that are currently supported by the system. For ease of reference, the templates have been divided into twelve broad topic areas.

4.2.1 Arithmetic: (13 templates)

BEDMASTemplate	Find a value of an arithmetic expression with brackets
HCFTemplate	Find a highest common factor of two numbers
WheelsRotationTemplate	Text question on highest common factor of several numbers
PrimesTemplate	Determine whether a given number is prime
PrimesCompositeTemplate	Four simple questions on prime numbers
SqrtTemplate	Find a value or simplify a square root
FractionTimesFractionTemplate	Find a product of two fractions
FractionDividedByFractionTemplate	Find a quotient of two fractions
FractionAddFractionTemplate	Find a sum of two fractions
FractionSubtractFractionTemplate	Find a difference between two fractions
FractionsTemplate	Evaluate an arithmetic expression with 4 fractions and brackets
PowerNegBaseTemplate	Evaluate an exponent (with a negative base)
AbsTemplate	Find an absolute value of a number or a simple expression

4.2.2 Algebra: (20 templates)

SqrtEquationTemplate	Find x from equation that contains square roots
InequalityToIntervalTemplate	Write an interval and mark it on a real line
IntervalToInequalityTemplate	Write an inequality and mark it on a real line
SqrtEquation1Template	Text question - find x from equation with a square root, x under the root
SimpleEquation1Template	Find a value of a variable from expression of type ax+b=c
SimpleEquation2Template	Find a value of a variable from expression c=ax+b
Expand1Template	Expand brackets in simple algebraic expression
Expand2Template	Expand brackets in more complex algebraic expression
AbsEquationTemplate	Find x in algebraic expression with absolute value, such as |ax +b|=c
SqrtMultiplyTemplate	Evaluate or simplify an expression with square roots, such as (sqrt(a)+sqrt(b))(sqrt(c)+sqrt(d))
SubstituteValueTemplate	Substitute values and solve an equation, for

	instance Let y=c, find x if y=ax+b
SqrtMultiply1Template	Evaluate or simplify an expression with square roots, such as (sqrt(a)+-sqrt(b))sqrt(c)
FractionEquation1Template	Find x if ax/b+c=d (x in numerator)
FractionEquation2Template	Find x if a/bx+c=d (x in denominator)
FractionEquation3Template	Find x if x=a/b +(-,*,/) c/d (x is not in fraction)
AddPowersTemplate	Simplify an algebraic expression with powers
SolveInequalityTemplate	Solve ax+b<cy+d
CancelPowersInFractionTemplate	Simplify an algebraic fraction
NumberProblems1Template	Solve a simple number problem of the form n+(n + 1) = a, where a is known
NumberProblems2Template	Solve a simple number problem of the form n+(n + b) = a, where a,b are known

4.2.3 Sigma notation: (7 templates)

FindXInSummationTemplate	Find x if x is in expression, i.e. sum{i=lowboudn}^{upperbound} ax = c
FindXInSummation2Template	Find x if x is a base of exponent in expression, i.e. sum{i=lowboudn}^{upperbound} x^i = c
EvaluateSummationTemplate	Expand and evaluate sigma notation sum{i=lowboudn}^{upperbound} (a^i)*i
ExpandSummationTemplate	Expand and simplify sum{i=lowboudn}^{upperbound} aix
EvaluateSummation2Template	Evaluat a sum in sigma notation, x=\sum{i=a}{b}{c\times i^{d}}
WriteAsSummationTemplate	Write an expression in sigma notation, for instance a*b+a*(b+1) + a*(b+2)... a*(b+k)
EquationInSummationTemplate	Find x in equation containing sigma, x could be in bounds, for example, \sum{i=x+a}{x+b}{c\times i}=d

4.2.4 Sets and probability: (4 templates)

SetIntersectionOf2Template	Find intersection of two sets and mark it on Venn diagram
SetOperationsOn2SetsTemplate	Composite question: union, intersection and difference of two sets
SetOperationsOn3SetsTemplate	Composite question: union, intersection, difference on three sets
ProbabilitiesTemplate	Composite question on probability

4.2.5 Straight lines, graphs, distances, intersections and simultaneous equations: (18 templates)

DistanceTemplate	Find the distance between to points
GradientInterceptSimpleTemplate	Find the gradient and y-intercept of the line ay+bx+c=0

GradientInterceptTemplate	Find the gradient and y-intercept of the line defined by more complex equation
LinearEquation1Template	Find the equation of the line with gradient m and passing through the point (x1, y1)
Line2PointsTemplate	Find the equation of the line passing through two given points (x1, y1) and (x2,y2)
LinearParallelSimpleTemplate	Find the equation of the line parallel to the given line (defined by a simple equation) and passing through the point (x1, y1)
LinearParallelEquationTemplate	Find the equation of the line parallel to the given line (defined by more complex equation) and passing through the point (x1, y1)
LinearPerpendicularTemplate	Find the equation of the line perpendicular to the given line and passing through the point (x1,y1)
LineThroughPointTemplate	Does the line ay+bx+c=0 pass through the point (x1,y1)?
LinePerpendicularToHorizontalTemplate	Find the equation of the line perpendicular to ay+c=0 and passing through the point (x1,y1)
LinePerpendicularToVerticalTemplate	Find the equation of the line perpendicular to ax+c=0 and passing through the point (x1,y1)
LineParallelToVerticalTemplate	Find the equation of the line parallel to ax+c=0 and passing through the point (x1, y1)
LineParallelToHorizontalTemplate	Find the equation of the line parallel to ay+c=0 and passing through the point (x1,y1)
SimultaneousEqnsSubstitution1Template	Solve simultaneous linear equations. Solution given is by substitution
SimultaneousEqnsElimination1Template	Solve simultaneous linear equations. Solution given is by elimination
SimultaneousEqnsElimination2Template	Solve simultaneous non-linear equations. Solution given is by substituting a non-linear term with new variable, and then solving resulting linear equations by elimination
SimultaneousEquationsLines1Template	Do two lines intersect? If they do, find the intersection point. Solution to the simultaneous equations is by elimination
SimultaneousEquationsLines2Template	Do two lines intersect? If they do, find the intersection point. Solution to the simultaneous equations is by substitution

4.2.6 **Functions, domain, range, quadratics and trigonometric functions:** (13 questions)

FunctionGraphsCompositeTemplate	Match each of 8 equations with its corresponding graph. Up to 20 different types of equations are given
Domain1Template	Find the domain of a simple function
Domain2Template	Find the domain of more complex function

Range1Template	Find the range of a simple function
Range2Template	Find the range of more complex function
DomainRangeTemplate	Evaluate the domain and the range for a given function
Quadratic1Template	Solve a quadratic equation using the quadratic formula
Quadratic2CompositeTemplate	Solve a number of equations without using the quadratic formula
Quadratic3Template	Solve an equation without using the quadratic formula
Quadratic4Template	Substitute a value of x to evaluate a quadratic function. Find f(n), if f(x)=ax^2+bx+c
TrigFunctionGraphTemplate	Sketch a graph of a simple trigonometric function, such as y=-sin2x
RadiansIntoDegreesTemplate	Convert given angles from radians to degrees
DegreesIntoRadiansTemplate	Convert given angles from degrees to radians

4.2.7 Logarithms and exponentials: (10 templates)

LogarithmsTemplate	Evaluate a number of simple logarithms, composite template contains 8 different logs
PopulationGrowthTemplate	Text problem on population growth (exponential growth)
RadioactiveDecayTemplate	Text problem on radioactive decay (inverse exponential growth)
CompInterestCompositeTemplate	A few simple problems on compounding interest
PopulationGrowthLog1Template	Question on exponential growth. Find growth rate
PopulationGrowthLog2Template	Another question on exponential growth. Find the population doubling time
ContCompInitialTemplate	Text problem on cont. compounding interest
ContCompInterestTemplate	Text problem - find the future value on continuously compounding account
ContCompFindtTemplate	Text problem – find the time of investment
PeriodicCompInitialTemplate	Text problem – calculate the initial deposit in periodic compounding schema

4.2.8 Derivatives and integration: (15 templates)

Derivatives1Template	Find the derivative of a linear function
Derivatives2Template	Find the derivative of a quadratic function
Derivatives3Template	Find the derivative of a polynomial function of degree n
Derivatives4Template	Find the derivative of a function containing trigonometric functions
Derivatives5Template	Find the derivative of a function containing

	logarithms
Derivatives6Template	Find the derivative of a function, which may contain square roots, logarithms, exponents and trigonometric functions
DerivativesCompositeTemplate	Composite question: find a derivative, solve an equation involving this, find a second derivative, find a derivative at a given point
DerivativesChainSimpleTemplate	Find a derivative of a composite function using the chain rule
DerivativesQuotientSimpleTemplate	Using quotient rule, find y', where y is a fraction with linear numerator or (and) linear denominator
DerivativesQuotientSimple2Template	Using quotient rule, find y', where y is a fraction with polynomial numerator or (and) denominator
DerivativesQuotientSimple3Template	Using quotient rule, find y', where y is a fraction with quadratic numerator or (and) denominator
DerivativesProductSimpleTemplate	Using product rule, find the derivative of a product of two simple functions
IntegralIndefiniteSimpleTemplate	Find the antiderivative of the function
IntegralDefiniteSimpleTemplate	Find the definite integral of the function
TrigDefiniteIntegrationTemplate	Find the definite integral of some trigonometric function

4.2.9 **Matrices:** (7 questions)

MatrixAddMatrixTemplate	Find a sum of two matrices
MatrixSubtractMatrixTemplate	Subtract one matrix from another
MatrixTimesMatrixTemplate	Find a product of two matrices
MatrixInverseTemplate	Find the inverse of a matrix
MatrixTimesNumberTemplate	Multiply matrix by scalar
MatrixExpressionTemplate	Evaluate an expression involving a few matrices
Sim3EquationsTemplate	Solve 3 simultaneous equations. Solution involves matrices

4.2.10 **Introductory programming skills with Python:** (9 templates)

AsCalculatorIntsTemplate	Evaluate a number of arithmetical expressions
AsCalculatorDistanceTemplate	Find distance between the two points – calculate it using Python
AsCalculatorExpTemplate	Evaluate an expression with exponents
AsCalculatorSinCosTemplate	Evaluate an expression involving trig functions
AsCalculatorTrigDegreesTemplate	Evaluate a number of trig functions
AsCalculatorLogsTemplate	Evaluate a number of logarithmic functions
PythonMatrixInverseTemplate	Find an inverse of a matrix
PythonSimultaneousEquationsTemplate	Solve simultaneous equations
ForLoopTemplate	Print a sequence of values, using loops

4.2.11 Science (examples from biology, physics, chemistry): (5 templates)

LeslieMatrixTemplate	Estimate the population at given times, using simple Leslie model
AvogadrosLawTemplate	Problem on Avogadro's Law
BoylesLawTemplate	Problem on Boyle's Law
IdealGasEquationTemplate	Problem on ideal gas equation
SurfboardTemplate	Problem on flotation of an object

4.2.12 Business and Finance: (17 questions)

AnnuityEasy1Template	Question on effective annual rate. Determine the periodic rate of interest given the EAR and number of periods
AnnuityEasy2Template	Another question on effective annual rate. Determine the annuity
AnnuityEasy3Template	Determine the number of periods
AnnuityIntermediate1Template	More complex question on effective rates
EffectiveAnnualRateEasy1Template	Calculate EAR of a nominal term
EffectiveAnnualRateEasy2Template	Make a decision on investment
EffectiveAnnualRateEasy3Template	Calculate a periodic rate
EffectiveAnnualRateIntermediate1Template	Calculate EAR of a nominal term – more complex problem
EffectiveAnnualRateIntermediate2Template	Make a decision on investment – more complex problem
EquityValuation1Template	Question on equity valuation. Find the price of a share
EquityValuation2Template	Find the equity return
EquityValuation3Template	Find the growth rate of the company
Utility1Template	Simple question on utility function. Calculate the utility of particular level of wealth
Utility2Template	Find the expected payoff and utility
Utility3Template	Find the expected payoff and utility on the fair coin game
Utility4Template	Make a decision on insurance premium
Utility5Template	Find an acceptable insurance premium

4.3 Sample output

Each of the templates described in Section 4.2 produces one or more questions of a particular type, along with the corresponding short answer (which is just the final answer arising from the calculation) and also fully worked solutions. Running the system multiple times produces random variants of each question, with different solutions. An example of a question, short answer and fully worked solution arising from each template is given in the attached document, and can be used by students and educators to tailor materials to their own specific learning requirements.

5. Factors impacting on the project.

A number of unanticipated factors had an impact on the project. These factors were mostly beneficial, but there were several lessons learned that would result in some changes if a similar project were to be undertaken again.

5.1 Factors ensuring success

The successful outcome to this project arose from a number of critical factors. Some were expected, including:

- A heavy reliance on the availability of skilled programmers with a high level of mathematical sophistication, and experience in helping students to learn mathematical and quantitative concepts
- Strong "buy-in" from academic staff to proof-read and amend resources produced by the system
- The value of seeking and responding to feedback from students about which aspects of the system were effective, and which needed to be reworked
- Administratively, having a motivated and participatory project team and reference group, and a budget that was reasonable for the nominated activities.

However, there were some unexpected factors that proved to be important to the success of the project, and enabled it to move beyond the initially anticipated scope. In particular, there were two external developments which each opened interesting and valuable opportunities.

First, in late 2006 Adams was awarded a joint Associate Fellowship of the Carrick Institute, with Professor Philip Poronnik at UQ, to work on embedding quantitative principles into life-science education.

Second, UQ undertook a comprehensive review of the BSc, a flagship degree with around 1000 students enrolling each year. For the first time, students in the UQ BSc will complete introductory courses with a heavy quantitative content, contextualised into a variety of scientific discipline areas such as biology, chemistry, psychology, physics and earth sciences. There is a lot of nervousness amongst teaching staff about how students will "cope" with quantitative content, and how UQ can provide the best-possible learning aids and assistance.

It was clearly beneficial for this project to be informed by, and in turn to influence, those developments, both of which had a strong mathematical and quantitative focus. As part of their fellowship, Adams and Poronnik identified exactly what quantitative knowledge life-sciences students need, what they typically have, and what aids they need to assist in learning such material. This then informed what happened in the compulsory quantitative courses, and what material must be covered. A new course was introduced in Semester 1 2008, taken by around 570 students, combining mathematical content with scientific applications and computer programming. Adams

coordinated and presented this course, and the system developed in this project was one of the key learning aids used in the new course. Indeed, when the application was originally submitted there was no intention for the system to provide support for computer programming exercises, but as a result of using the system in the new course, nine Python programming templates have been developed, and it is intended that more will be written.

Two additional factors increased success of the project. First, it became clear that there is a potential opportunity to include components of the system in a comprehensive electronic learning aid for students in business finance. This is still being pursued, and 17 relevant question templates have been developed. Second, additional development money has been provided by The University of Queensland to increase the depth and breadth of resources available in the system. Development is continuing, and this is discussed below.

Finally, a significant strength of this project is that it had a reasonable scope. The project was deliberately well-defined, with a specific goal of producing a software system, so there was no tendency to become sidetracked, or to discover belatedly that the scope of the project was too broad. However, even with a comparatively clear project plan, a number of useful extensions were identified. The project is continuing, with additional funding, to develop these extensions, and the outcomes are stronger because of this.

The key lesson for other projects learned from this final point is the importance of having a fairly well-defined project as far as possible, with an appropriate scope that does not aim to solve all problems associated with the area of work. Ideally, any project should also identify potential sources of future funding to help ensure sustainability and further work on interesting extensions that are identified during the project activities.

5.2 Factors impeding success

A number of factors impeded the project, although the negative impact was mostly reflected by a delayed timeline rather than a reduced final outcome. The following points are noteworthy, and in each case the key lesson is identified. (Of course, some of these lessons are generic, and already well known.)

- **Delays in appointing skilled staff:** The initial proposal highlighted the critical role to be undertaken by a software engineer with experience in software development, mathematics and mathematical typesetting. However, we did not anticipate the difficulty we would encounter in finding a suitable candidate. Enrolments in IT courses have collapsed over recent years, and it also became apparent that many of the top students have changed their course selections to other areas. Selection was made much more difficult by our requirements for the appointee to also have a reasonable level of mathematical sophistication. As a result, a number of candidates were either judged as being not suitable, or declined appointment. After a substantial delay we were fortunate to attract an expert in both mathematics and

computing, but this extended the timeline. There is no doubt that this was an excellent appointment, and the person continued to work on the extended project.

The key lesson learned from this point is to allow more time than expected to appoint key, skilled staff.

- **Over-commitment of members of project team:** This is certainly not an uncommon experience, but key members of the project team (particularly Adams) became involved in too many additional activities, related or unrelated to this project. Effective mechanisms were established within the project to ensure that the technical work always continued in a very organised and timely manner, but no such arrangements were made with project reporting requirements. As a result, a number of important and reasonable administrative and reporting deadlines were not met. So the technical aspects of the project proceeded in an excellent manner and the outcomes are strong, but the failure to meet other deadlines was clearly very far from satisfactory.

  The key lesson from this point is to attempt to ensure that key participants in a project are not overloaded with extraneous activities, or, if they are, to ensure that an effective mechanism is in place to meet ALTC deadlines. (Of course, mechanisms must also be in place to ensure that project work is undertaken, but that happened for this project.)

- **Some components took an unexpectedly long time**: Developing and debugging some of the templates was much more time consuming than was anticipated. For example, the mathematically simple concept of operating with fractions proved to be the most demanding template to implement, as there are so many special cases and ways of solving the problems that do not "feel right". It took around 2 months of full time work to complete this section.

  The key lesson learned from this point is that some tasks in a project that might appear to be fairly well-defined and uncomplicated will in fact prove to be a stumbling block. Initial analysis and preliminary investigations may help to identify accurately where more time is likely to be required, but unexpected factors will probably arise.

6. Applicability to other institutions/locations.

This system is designed to provide learning assistance to students studying mathematical and quantitative concepts. There is broad international agreement on the specific technical knowledge in these areas required by students at certain stages of their education. Particular equations or problem-solving techniques are studied universally, precisely because they represent many real phenomena and because they need to be analysed and solved in specific valid ways.

Two stated goals for the original system were to include substantial breath of content, and to have broad applicability. These goals have both been attained. The system was designed for students at the early-tertiary level (and hence also links closely to the upper-secondary level). Thus the concepts and content included in the system are of direct relevance and applicability to any students in Australia (and indeed internationally) who are transitioning from secondary to tertiary education.

One of the largest growth areas in mathematics enrolments in Australian Universities is in courses that cover intermediate and advanced senior secondary mathematics, and most Universities offer these transitional courses. The current system has been used extensively in one such course at UQ, and has proved to be effective. Thus the materials within the system would be of direct use in almost any University or secondary school in Australia or internationally.

In addition to students studying mainstream mathematics courses, the system is also intended to be used by students in other areas including business, economics, engineering, chemistry and biology. A number of the existing question templates cover those topics, and so the system is useful for students working in those areas.

However, before the system will be more widely adopted, additional work is required on the system interface and appropriate user documentation. This work is underway, and will be completed later this year. In summary, the current materials are certainly usable and will continue to be used in the UQ transitional mathematics courses, but usage will increase when the system interface is more “user friendly”.

7. Dissemination.

The results of this project have been or will be shared via the following mechanisms:

- A conference presentation on the system developed in this project was made in 2007 at an international conference on virtual techniques in education:

  A New Enabling Technology for Teaching and Learning Quantitative Skills, Adams, P., Kvyatkovskyy, A., Zinchenko, M., Proceedings of the 11th Annual International Conference on Education and Virtuality (2007), Ed: Grebenyuk,V.A., Kinshuk, Semenets, V.V., pages 282 – 291.

- Following the above conference presentation, a member of the project team was invited to make a presentation at the Department of Applied Mathematics and Informatics in Donetsk National Technical University, in Ukraine. A copy of the system as it currently stood was left with them, and they expressed an interest in using the system in their introductory informatics and programming classes this year, and looking at developing new modules and question templates.

- We intend to make a presentation at a conference organised by the Australian Association of Mathematics Teachers within the next 12 months, with the audience composed primarily of secondary school teachers.

- As discussed below, this project has close links with the Carrick Associate Fellowship of Adams/Poronnik, so the system arising within this project will be discussed briefly in presentations associated with the Fellowship.

- The main developments currently being undertaken are strengthening and standardising the system interface, building a comprehensive web-based front end. Once this is complete, the web-based interface will “go live”, and will be made accessible to any interested users and contributors. The webpage will be advertised on UQ web pages, at mathematics and education conferences, on ALTC Discipline Community websites and also on pages of professional mathematics societies.

- One possibility for further dissemination that has arisen during the project is incorporating business and finance questions on an electronic resource that is already distributed broadly within Australia to several thousand students each year. This is still being investigated, but we are hopeful that this will prove possible.

8. Links with other ALTC projects.

In late 2006 Adams was awarded a joint Associate Fellowship of the Carrick Institute, with Professor Philip Poronnik at UQ, to work on embedding quantitative principles into life-science education. The key research questions guiding that fellowship are to investigate: what are the real levels of mathematical skills possessed by, and required by, first year Life Science students; what are the most appropriate methods of assessing the levels of Life Science student engagement with mathematical concepts; and how can essential mathematical principles be embedded in the context of first year Life Science education? The broad aims are to:

(1) Research and develop context-sensitive instruments to determine the level of mathematical and quantitative competency achieved and required in first year Life Science students and their engagement in the quantitative aspects of research in the Life Sciences; and

(2) Initiate the development and implementation of transferable and scalable teaching tools that enable the embedding of relevant mathematical principles within Life Science courses.

There is a strong link between Goal (2) of the Fellowship and the system developed in this project. An important part of the learning process for students of quantitative life sciences (as in other quantitative areas) is access to relevant, accessible practise materials, such as those developed in this project. Conversely, the Fellowship activities have played an important role in identifying the key quantitative skills with which students in life science typically encounter difficulties, and how these concepts can best be illustrated in specific examples. Thus the fellowship and this project have each both informed, and been informed by, activities in the other.

On a less formal level, this project has benefited from suggestions and ad hoc discussions held with other Carrick Fellows, including Helen MacGillivray, Merrilyn Goos and Michael Bulmer, all of whom have significant expertise in quantitative education processes, and with Ron Oliver, who has expertise in the dissemination and uptake of computational resources.

Finally, there is a strong link between this project and the goals of the Discipline-based initiative in teaching mathematical concepts to engineering students, which was recently completed. That initiative has identified a range of relevant materials, which will be collated into a comprehensive resource for use with engineering students entering their first year of studies. Once the remaining development phases of this project are completed later in 2008, the web interface for this system will be linked into the engineering resource.