Overcoming algebraic misconceptions that inhibit students' progress in mathematical sciences

Final Report 2014

The University of Melbourne

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List of acronyms used

SMART	Specific Mathematics Assessments that Reveal Thinking
MGSE	Melbourne Graduate School of Education
UoM	The University of Melbourne

Executive summary

Declining tertiary enrolments in post compulsory/core mathematical sciences together with the high abandonment rate from one year to another is a major national concern (Chubb et al., 2012). This seed project, funded by the Officer for Learning and Teaching (OLT), made a sound start in addressing this important issue by providing strong evidence of fundamental mathematical misconceptions that persist even in apparently successful secondary students.

The project focused on two core mathematical concepts that are known to be part of the essential foundation for success in tertiary mathematics: variables and functions. Preliminary data collection found that most students correctly carried out procedures, most had difficulty explaining in plain English and as many as 20 percent showed evidence of fundamental misconceptions related to functions or variables. These findings from students who may be expected to be in the top 10 percent of their senior secondary cohort suggest that these issues will be greater at some other universities. As mentioned in the rationale for our project, the availability of Computer Algebra Systems (CAS), especially in examinations, has been controversial and it was anticipated that there may have been some evidence of the impact of its influence on students' conceptual understanding and algebraic skills. A background survey was specially designed to investigate students' use of CAS at Year 12 and information showed that 60 percent of students had used CAS in their Year 12 mathematics examinations. However, statistical analysis showed almost no significant association between CAS availability and the mathematical skills and concepts tested. The one exception was that, presented with a graphical representation, students who had used CAS were more likely to correctly identify functions. We have not discussed the non-significant results in this report.

Informed by the evidence of student misconceptions the project designed two innovative diagnostic assessments based on Specific Mathematical Assessments the Reveal Thinking (SMART tests). These quick online tests not only provide teaching staff with a detailed summary of their students' results but also identify key misconceptions and provide advice for appropriate teaching strategies. This seed project showed that students are often not aware of the misconceptions they hold (specifically because they have successfully progressed through secondary school), but, moreover, university tutors had not anticipated the fundamental flaws in their students' thinking.

The outcomes of this project suggest that students' mathematical foundations may be reinforced by identifying incomplete conceptions and addressing persistent misconceptions, which in turn should increase student retention and timely progression in mathematical sciences. Strengthening foundations would require a suite of these online resources covering a wide range of key topics. For students to obtain full benefit from undertaking a number of these online assessments the SMART test format should be enhanced to provide reports and guidance directly to students.

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Approach and methodology

The key feature of the project was the effective use of online technology to test students and provide immediate feedback through the development of SMART tests. Up to now, these tests were only used in secondary schools and trialed with pre-service primary school teachers. During this seed project, through collaboration of research and teaching staff from mathematics and mathematics education, two key topics identified in the literature as a source of difficulty for tertiary mathematics students, were explored: variables and functions. The methodology followed a three stages process.

Stage 1

In stage 1, the project developed an evidence base for incorrect or incomplete fundamental algebraic conceptions prevalent amongst commencing first year undergraduate mathematics and statistics students.

This data was collected from 427 volunteer first year undergraduate mathematics and statistics students at the UoM via a timed screening (35 minutes) mathematics quiz (*appendix 1*) made available through the university's Learning Management System (LMS) at the beginning of first semester. Students who participated in this study were undertaking at least one of the following subjects: Calculus 1, Calculus 2, Accelerated Mathematics 1, Experimental Design and Data Analysis, Introduction to Mathematics and Mathematics for Biomedicine. A range of students who responded to the mathematics quiz where then invited later on during the semester for individual 'think aloud' interviews, which provided richer and deeper information about the mathematical concepts students held.

In parallel data was collected regarding students' study of mathematics in their senior secondary years (665 students provided at least some responses). This was done using an online survey (*appendix 1*) made available at the same time as the screen test referred to above. Technical constraints of the LMS mean that this was best presented as two separate tasks. The survey covered basic demographic items then some detailed questions regarding students' perceptions of both their teachers', and their own, use of technology for learning mathematics.

Students' attention was drawn to the quiz and survey through emails and verbally by their lecturers. Findings published (and forthcoming –see Outcomes) were based on 383 students who both answered the mathematical quiz and the background survey.

Stage 2

Evidenced by the data collected in stage 1, stage 2 aimed to develop examples of diagnostic SMART tests appropriate to the needs of the students. This involved imaginative creation of items grounded by research literature, statistical and qualitative analysis of student responses to the items, statistical implicative analysis to identify underlying problems, followed by item and test trialing.

Stage 3

Finally, stage 3 consisted of developing immediate online feedback for tutors describing appropriate teaching strategies in response to the identification of these specific algebraic misconceptions.

Outcomes

- An evidence base that identifies incorrect and incomplete key mathematical conceptions that first year university students hold in the area of variables and functions.
- Although 'functions' are a key topic in the secondary school mathematics curriculum, we found that students had difficulty defining/describing the notion of function in plain English (Question 1 of the mathematics quiz - *appendix 1*). Although the majority of students from our study (61 percent) did provide at least one valid statement, only one or two were comprehensive. The students' responses to the related other questions suggested that their descriptions did not match their understanding. However, the results for questions focused on different aspects of a function (graphical, symbolic, numeric, etc) which suggests that students do not perceive the links between its multiple representations. And certainly the often over-emphasised 'teaching tool' of the 'vertical line test' (a vertical line drawn anywhere on the graph of a function crosses the graph only once) is apparently of little help – probably because it has been memorised rather than meaningfully understood. It is surprising how closely our results match with previous research, despite the difference in schooling systems, the decades that separate our study from previous studies and the availability of technology, especially graphics calculators, that may be used pedagogically to develop cross representational schema (for detailed results see: Bardini, Pierce & Vincent, 2013).
- The mathematics quiz questions related to variables showed that a small but concerning number of students apparently have incorrect or incomplete conceptions of the use of letters in mathematics. The proportion of students with these misconceptions in our study is not large (maximum error rate was 25 percent for the item "When is the equation $L+M+N = L+P+N$ true? Always, Never, Sometimes") but it should be kept in mind that these are students who have successfully passed through the school system with sufficiently high scores in their final year of school to be accepted into first year mathematics at a high profile university. It is not unreasonable then to hypothesize that if the study were conducted in first year tertiary mathematics units across the country, the proportion of students with these misconceptions would be much higher (for detailed results see: Bardini, Pierce, Vincent, King, 2014)

  While two-thirds of the UoM mathematics students surveyed had used computer algebra systems for their Year 12 mathematics, the only evident link to their conceptual understanding of functions and variables was that those who had used this technology showed a stronger grasp of graphical representations of functions. Students' reported perceptions of their teachers' use and their own use of this technology (as measured by the survey items) suggest that this technology is largely used for answer checking and speed rather than as an additional pedagogical tool.
- Two new SMART tests suitable for tertiary students. This on-line format seemed to be convenient for students and teaching staff, whilst also drawing a more sophisticated picture of a students' understanding of mathematics than can be time-efficiently produced by hand. The main SMART test system is accessible through

<www.smartvic.com/smart/index.htm>

At present the two pilot tests for tertiary students may be accessed by emailing the SMART test office using the contact details given on the SMART test website.

Online feedback to staff, illustrating appropriate teaching strategies and examples for use with students who are identified as having specific misconceptions. This feedback is linked to the tests which may be accessed as indicated above.

- A fully refereed conference paper:

  Bardini, C & Pierce, R., Vincent, J. (2013). First year university students' understanding of functions. Over a decade after the introduction of CAS in Australian high schools, what is new? In L. Rylands, B. Loch & D. King Proceedings of the 9th Delta conference on teaching and learning of undergraduate mathematics and statistics. pp2-11. Available online from: <delta2013.net/documents/ConferenceProceedings.pdf>

- Established collaboration between mathematics education and mathematics/statistics staff at UoM.
- Proposals for further projects:
  - o The University of Melbourne has provided a small grant so that the project can continue to develop the collaboration between Mathematics Education and Mathematics and conduct pilot research on university students' fundamental understanding of mathematical symbols. This project is titled "Secondary and university mathematics: do they speak the same language?" (see below).
  - o The team have also developed a proposal for submission to the Australian Research Council Discovery grants program. This project is entitled "Secondary and university mathematics: do they speak the same language?" and would extend the limited pilot work we are able to undertake as part of the locally funded research. In summary, this project aims to examine students' experience of learning mathematics through the lens of the use of mathematical symbols. Mathematics at university is highly symbolic. The project will test whether this is a major difficulty, and if so, identify the major dimension of the problem and examine impact on progression rates. Outcomes of the project include illustrated description of similarities and contrasts between conventions and use of symbols in secondary and university mathematics and across different mathematical sciences. It will also provide evidence of links between students' response to increased symbolic load and their confidence to continue studying subjects with high mathematical content at university.

  Three further articles are in process including:

  Bardini, C., Vincent, J., Pierce, R. & King, D. (2014). Undergraduate mathematics students' pronumeral misconceptions. SUBMITTED for the *Mathematics Education Research Group of Australasia's* annual conference, Sydney: July, 2014.

Note: Evaluation of the SMART tests with tertiary students has been limited because they are designed to be most effective when used at the point of transition from secondary school. It is envisaged that they will be used more extensively in semester 1, 2014

Recommendations

- The two SMART tests developed as part of this project be trialled more extensively with students and their tutors.
- SMART test feedback for tertiary students be further refined so that personalised learning advice may be provided, according to the particular stage of learning and misconceptions that the student is diagnosed to have, in addition to teaching advice for tutors. This would allow the option of students using such resources autonomously.
- The evidence base for tertiary students' persistent fundamental misconceptions across the range of key areas of mathematical understanding that tertiary staff take to be assumed knowledge is developed.
- Further SMART tests providing diagnosis and teaching and learning advice for staff and students are developed.

Dissemination

Preliminary findings from the project were presented, six months after the beginning of the project, at the 9th Delta conference on the teaching and learning of undergraduate mathematics and statistics (Southern hemisphere conference). This led to the publication of a fully refereed conference paper.

At the end of the project, the project team presented the details of the methodology and outcomes at The UoM Department of Mathematics and Statistics seminar with a threefold purpose: share the findings from data collection forming evidence base for the resource developed, demonstrate the resource-online diagnostic tests and report findings to date of trial of the resource.

Linkages

Through this project an interdisciplinary linkage has been established between mathematics and statistics in the science faculty and mathematics/statistics education in the Graduate School of Education. This link has not been operational in the past but has now begun and staff from both areas see mutual benefit in undertaking further projects together to respond to the urgent need in Australia to encourage the study of mathematical sciences. The project draws on the expertise of the MGSE Mathematics Education group in online diagnostic testing and the expertise of mathematics and statistics tutors in noting topics which cause students difficulties.

Evaluation

The SMART test resources developed are designed identify and offer remedial strategies to support students' transition from school to university mathematics. While a small number of students accessed the trial tests late in semester 2, 2013, many more students will be directed to them at the beginning of semester 1 2014. It is therefore the intention of the project team to conduct further evaluation based on this experience.

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Appendix 1 Background Survey and Mathematics Quiz

The survey and quiz were made available through the University's Learning Management System. The layout therefore was not the same as that provided below. Students were emailed to alert them to this task and to provide the Human Research Ethics Committee approved Plain Language Statement describing the project.

Survey & Quiz
Please click on the link to a survey to collect information regarding your background experience and a quiz probing your understanding of some key mathematical ideas. This information will help your lecturers and tutors be more attuned to your needs and this data will also contribute to a research project (details provided on the linked site).
We expect that the background survey should take about 5 minutes to complete. The key mathematics ideas quiz is time restricted to 30 minutes and must be done in one sitting. You should make choices and move on quickly. You may like to have pen and paper available for small calculations. Please do not use a calculator as this would not help us help you.

Link to background survey and maths quiz

Background Survey – use of mathematics technology

Q1. How old are you?

Age ≤20

20< Age ≤25

25< Age≤35

Age >35

Q2. Where did you undertake your final year of secondary education in Victoria?

VICTORIA

Other - please specify name of State or Country___________________________

Q3. Which Year 12 (VCE, Unit 3/4) mathematics subject(s), or alternative course did you study (choose as many as apply)

Further Mathematics

Mathematics Methods

Specialist Mathematics

Other subject(s) (please specify) ____________________

____

Q4. During your year 12 mathematics studies what technology, if any, did you use and where? Choose as many as apply.

	Not used	Used in class only	Used out of class only	Used in and out of class	Used in final exams
CAS calculator (e.g. Nspire(CAS) , Classpad)	o	o	o	o	o
Graphics calculator	o	o	o	o	o

Scientific Calculator	○	○	○	○	○
Arithmetic calculator	○	○	○	○	○
Computer based CAS (e.g. Nspire or Classpad on a laptop, Mathematica)	○	○	○	○	○
Computer based spread sheet (e.g. Excel)	○	○	○	○	○
Other – please specify technology and use					

For the next 12 items, think about YOUR TEACHER'S USE, in class, of mathematics technology . Did they use it for:

	Not used	Hardly ever	Occasionally in a few topics	Occasionally in most topics	Often in a few topics	Often in most topics
Checking answers	o	o	o	o	o	o
Obtaining results faster than with pen & paper	o	o	o	o	o	o
Doing calculations that students might find hard	o	o	o	o	o	o
Doing algebra that students might find hard	o	o	o	o	o	o
Doing application problems	o	o	o	o	o	o
Showing the impact of varying coefficients, powers etc.	o	o	o	o	o	o
Creating tables	o	o	o	o	o	o
Graphing functions	o	o	o	o	o	o
Solving	o	o	o	o	o	o
Expanding or factorising	o	o	o	o	o	o
Differentiating or integrating	o	o	o	o	o	o
Doing matrix operations	o	o	o	o	o	o

For the next 12 items, think about YOUR OWN USE of mathematics technology. Did you use it for:

	Not used	Hardly ever	Occasionally in a few topics	Occasionally in most topics	Often in a few topics	Often in most topics
Checking answers	o	o	o	o	o	o
Obtaining results faster than with pen & paper	o	o	o	o	o	o
Doing calculations that I find hard	o	o	o	o	o	o
Doing algebra that I find hard	o	o	o	o	o	o
Doing application problems	o	o	o	o	o	o
Showing the impact of varying coefficients, powers etc.	o	o	o	o	o	o
Creating tables	o	o	o	o	o	o
Graphing functions	o	o	o	o	o	o
Solving	o	o	o	o	o	o
Expanding or factorising	o	o	o	o	o	o
Differentiating or integrating	o	o	o	o	o	o
Doing matrix operations	o	o	o	o	o	o

Mathematics Quiz

Question 1

Explain, in plain English, what a function is.

____

____

____

____

Question 2

Which of the following rules define a function? (Mark "X" in front of the rule. Choose as many as apply)

$f(x) = x^2 - \sqrt{2}$ where $x \in \mathbb{R}$

$f(x) = ax + b$ where x, a, b $\in \mathbb{R}$

$$f(x) = \begin{cases} x^2 & if\ x \leq 0 \\ 2 & if\ 0 < x \leq 1 \\ 2x & if\ x > 1 \end{cases} \qquad \text{for all } x \in \mathbb{R}$$

$$f(x) = \begin{cases} 2x + 1 & if\ \ x < 2 \\ -5x + 3 & if\ \ x > 1 \end{cases} \qquad \text{for all } x \in \mathbb{R}$$

Let f be a function whose rule $f(x)$ is the area of a circle with circumference x

Let f be a function whose rule $f(x)$ is the area of a rectangle with perimeter x

Question 3

Let f be a function such that, for all real x, $f(x) = x^2 - x + 2$.

a) What is the value of $f(1)$?

____

b) Find $f(a)$. Find $f(-a)$.

____

____

c) Find $f(2x)$, $f(-x)$, $f(x+1)$ and $f(x^2)$.

____

____

____

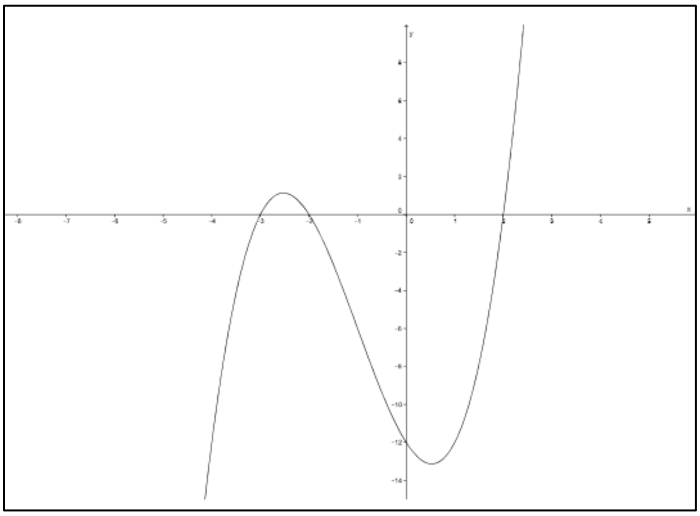
____

d) What is the coefficient of x^2 in $f(f(x))$? (Mark "X" in front of your choice)

1. 0
2. 1
3. 2
4. 3
5. 4

Question 5

The graph of one of the functions is shown. Which function must it be? (Mark “X” in front of your choice)



a) $f(x) = 4x - 12$

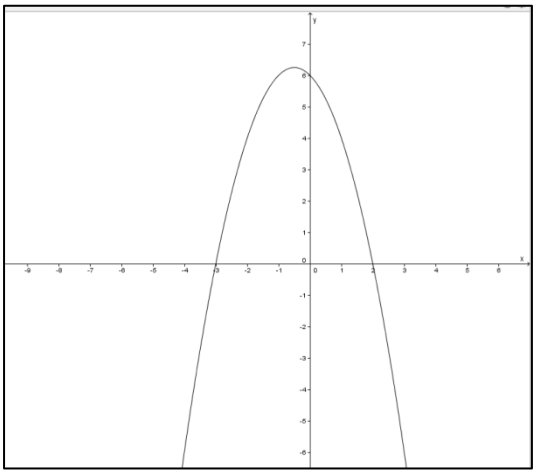
b) $f(x) = 2x^2 + 3x - 2$

c) $f(x) = e^x - 12$

d) $f(x) = x^3 + 3x^2 - 4x - 12$

Question 6

The graph of one of the functions is shown. Which function must it be? (Mark “X” in front of your choice)



a) $f(x) = 6 - x$

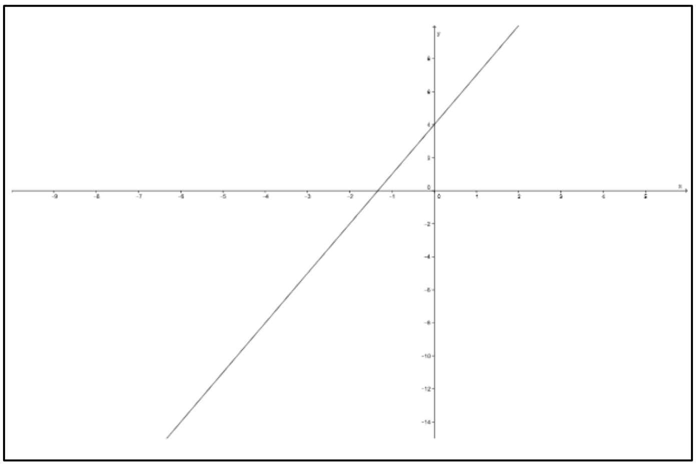
b) $f(x) = 6 - x - x^2$

c) $f(x) = 6 - 2e^x$

d) $f(x) = x^5 + 6$

Question 7

The graph of one of the functions is shown. Which function must it be? (Mark "X" in front of your choice)



a) $f(x) = 3x + 4$

b) $f(x) = 4 - x^2$

c) $f(x) = e^x - 2$

d) $f(x) = 2x^4 - 3x + 4$

Question 8

For the table below, the rule for the function which links *x* to f(*x*) could be (Mark "X" in front of your choice)

x	*f*(*x*)
-2	4
-1	1
0	0

1	1
2	4
3	9

a) linear

b) quadratic

c) exponential

d) none of the above

Question 9

For the table below, the rule for the function which links *x* to f(*x*) could be (Mark "X" in front of your choice)

x	*f*(*x*)
-2	0.04
-1	0.2
0	1
1	5
2	25
3	125

a) linear

b) quadratic

c) exponential

d) none of the above

Question 10

For the table below, the rule for the function which links x to f(x) could be (Mark "X" in front of your choice)

x	$f(x)$
-2	-5
-1	-4
0	-3
1	-2
2	-1
3	0

a) linear

b) quadratic

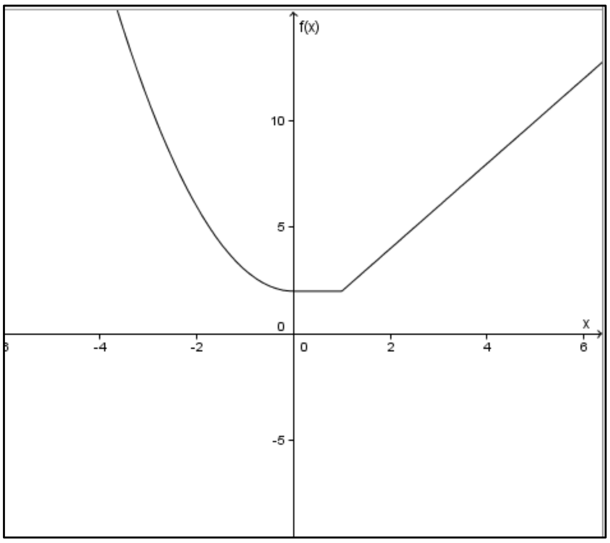
c) exponential

d) none of the above

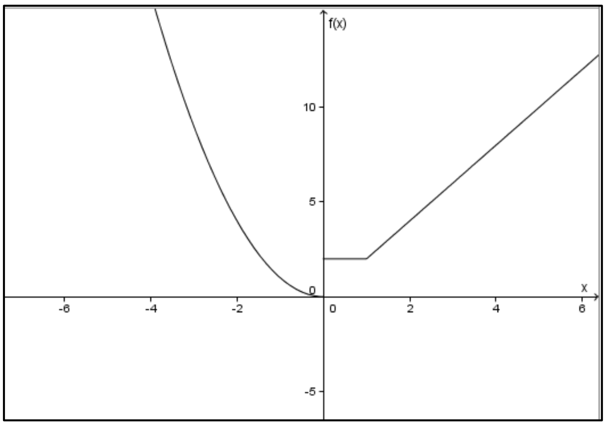
Question 11

Among the following graphs, which one corresponds to a function (Mark "X" in front of each name of the graph of your choice. Choose as many as they apply)?

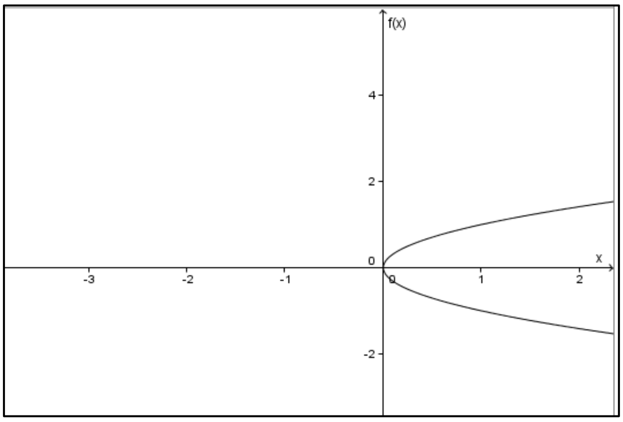
Graph 1



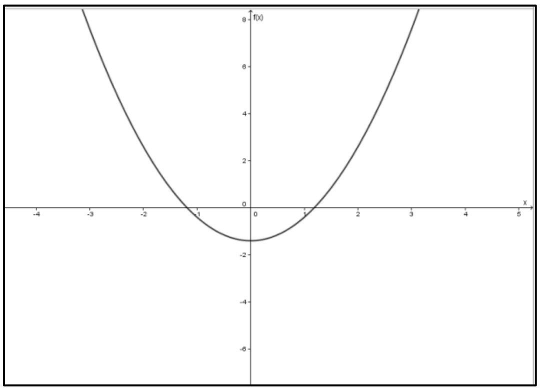
Graph 2



Graph 3

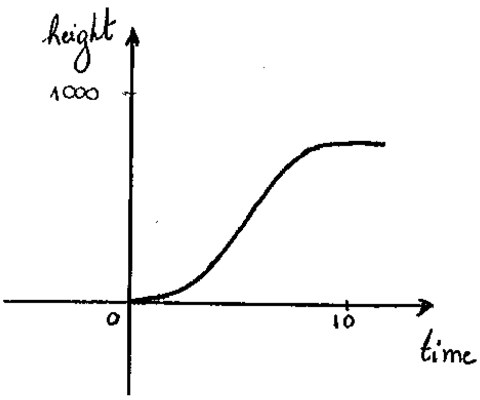


Graph 4



Question 12

Below is the sketch a graph of the height of an aeroplane (in meters) against time (in minutes) since the airplane has left the terminal building.



Part a
Fill the blanks with <u>only one</u> of the following: increasing/decreasing/constant/none of these

For the graph as shown between $t = 0$ and $t = 10$:

a) The height is ______________ with time
b) The rate of change of height is _______________
c) The values of the function shown in the graph are ______________ with time
d) The first derivative is _______________

Part b
Fill the blanks with <u>only one</u> of the following: always positive/always negative/zero/none of these

For the graph as shown between $t = 0$ and $t = 10$:

a) The height is ___________________
b) The rate of change of height is ______________________
c) The rate of change of the rate of change of height is _____________________
d) The first derivative of the function shown in the graph is ___________________
e) The second derivative of the function shown in the graph is __________________

Question 13

When is the equation $L+M+N = L+P+ N$ true? (Mark "X" in front of your choice)

a) Always

b) Never

c) Sometimes

Question 14
Let N be the length of the Niger River in kilometres and let R be the length of the Rhine River in kilometres. Which of the following equations says that the Niger River is 3 times as long as the Rhine? (Mark "X" in front of your choice)

a) N=3R

b) R=3N

c) R=4N

d) N=4R

Question 15

Some students had to find some values of x to make this equation true:
$x + x + x = 12$

Select each student whose answer is correct (Mark “X” in front of your choice. Choose as many as apply).

a) Mary wrote $x = 2$, $x = 5$ and $x = 5$

b) Millie wrote $x = 9$, $x = 2$ and $x = 1$

c) Mandy wrote $x = 4$

Question 16

Some students had to find some values of x and y to make this equation true:
$x + y = 16$

Select each student whose answer is correct (Mark “X” in front of your choice. Choose as many as apply).

a) John wrote $x = 6$ and $y = 10$

b) Jack wrote $x = 8$ and $y = 8$

c) James wrote $x = 9$ and $y = 7$

Question 17

Noah sells k muffins. Maria sells five times as many muffins as Noah. They sell the muffins for $1.25 each.

The number of muffins Noah sells is a variable.

Name another variable in this problem. Please explain________________________________
____
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Name something in the problem that IS NOT a variable ________________________________