

Mathematics Education for 21st Century Engineering Students

Final Report

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Philip Broadbridge
Simi Henderson

Australian Mathematical Sciences Institute

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MATHEMATICS FOR 21ST CENTURY ENGINEERING STUDENTS

Universities involved

The Australian Mathematical Sciences Institute (AMSI) has invited participation from all 32 Australian universities with accredited engineering degree programs.

Australian Defence Force Academy	Swinburne University of Technology
The Australian National University	University of Adelaide
Central Queensland University	University of Ballarat
Charles Darwin University	University of Melbourne
Curtin University of Technology	University of Newcastle
Deakin University	University of New South Wales
Edith Cowan University	University of Queensland
Flinders University	University of Sydney
Griffith University	University of South Australia
James Cook University	University of Southern Queensland
La Trobe University	University of Tasmania
Macquarie University	University of Technology Sydney
Monash University	University of Western Australia
Murdoch University	University of Western Sydney
RMIT University	University of Wollongong
Queensland University of Technology	Victoria University

Initiative Leader

Professor Philip Broadbridge, Director, AMSI
111 Barry Street, University of Melbourne VIC 3010
T: 03 8344 1779 E: phil@amsi.org.au

Team Members

Research Assistant: Simi Henderson, AMSI
Project Manager: Graham Keen, AMSI
Admin Assistant: Parvin Ahadi, AMSI

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Executive Summary

The body of engineering students has been diversifying to such an extent that it has been raising new challenges, both for the students and for educators. This scoping project has been an important stock-taking exercise, reinforcing the links among mathematics and engineering educators and enabling us for the first time to form a picture of a rapidly changing landscape. Our study has been informed by a literature review, a survey questionnaire, on-site visits, and a one-day workshop.

Compared to 20 years ago, there is now much wider variability among incoming students in the level of mathematical competence. Mathematics educators have been trialling and adopting a variety of strategies to engage the students and to help them succeed. However, these adaptations have been made at the local level, with very little oversight and coordination at the national level.

The first adaptations have been structural, including various forms of additional developmental courses, streaming, and drop-in tutorial centres. Computer-aided assessment tools have helped to provide formative diagnostic assessment to large classes. Some mathematical software is used more universally than we had anticipated, giving scope for centralised development of teaching materials.

In order to accommodate topics in professional practice, the number of mathematics subjects has been reduced. This has necessitated the removal of some mathematics topics from the compulsory part of the curriculum but there is widespread disagreement on which topics should have the lowest priority. Compared to 20 years ago, it is now less likely for a 4-year BE graduate to be extensively trained in mathematics. The niche for mathematically strong engineers is being populated by a relatively small number of double degree students. These students are important as they make a significant contribution to Australia's mathematical capability.

There is widespread agreement that engineers need some mathematics in their training. With goodwill between the disciplines and some coordinated developments, we believe that engineering mathematics can be made more appealing. This could include more reference to engineering contexts in mathematics lectures.

After analysing our findings, we make recommendations on measures to cap the broadening diversity, to better manage the mathematical education of the current student group, to promote better collaboration between the disciplines, to provide a pathway to produce a minor stream of mathematically well qualified BE graduates and to provide a better service to meet the professional needs of the students.

The project provides an excellent model for future collaboration between mathematics educators and other disciplines. We would identify the mathematics education of biological scientists and teachers as priorities.

Recommendations

Recommendation 1: assumed knowledge

That engineering programs should continue to state that students will be assumed to have knowledge of material covered in Year 12 Intermediate Mathematics, including some calculus. For those students entering without that knowledge, an additional developmental subject must precede the normal mathematics subjects.

Recommendation 2: designated quantitative stream

That in 4-year BE programs with a first-year intake of 140 or more, 15% or more of the places be reserved for a designated quantitative stream in which students must take at least 5 subjects of mathematics, statistics, theoretical computer science, quantitative finance and theoretical physics.

Recommendation 3: statistics and stochastic modelling

That a single one-semester optional subject in statistics and stochastic modelling be made available to all engineering students who have completed three mathematics courses, if not already included in the syllabus.

Recommendation 4: joint mathematics curriculum committees

That every engineering program has a joint mathematics curriculum committee that is responsible for determining mathematical topics to be covered. The committee should meet at least twice per year and it should have representatives from engineering, mathematics and statistics departments, as well as two students who have recently completed some engineering mathematics subjects.

Recommendation 5: collaborative teaching

That universities modify their internal financial allocation system so that no budgetary unit is penalised for taking part in genuine multidisciplinary collaborative teaching.

Recommendation 6: engineering mathematics staff expertise

That mathematics departments in BE or ME-awarding institutions should identify which of their staff, if any, have knowledge of engineering applications. If this expertise is lacking, some future academic job advertisements should say that ability to teach mathematics in engineering contexts would be an advantage.

Recommendation 7: on-line formative assessment

That mathematics departments, assisted by the Australian Mathematical Sciences Institute and Australian Association for Engineering Education, investigate the introduction of automated systems of test generation, automatic marking and feedback, so that they can run compulsory on-line quizzes during semesters for large engineering mathematics classes.

Recommendation 8: collaborative item bank

That engineering and mathematics teaching departments collaborate to provide a central bank of good examples of formative test questions, computer laboratory projects and curriculum resources.

Recommendation 9: student help centres

That Engineering Faculties designate at least 4 common hours per week of class free time spread over 3 or more days and that servicing mathematics departments provide staff or senior students in student help centres at those times.

Recommendation 10: boosting senior secondary school mathematics

That able students be more strongly encouraged to progress to subjects comparable to Intermediate Year 12 Mathematics of New South Wales and Victoria.

Contents

Recommendations	iii
1. Initiative Description	1
2. Investigation Strategy	2
2.1 Advisory Committee	2
2.2 Literature Review	3
2.3 Site Visits	4
2.4 Questionnaire	4
2.5 National Symposium	4
2.6 Terminology	4
3. Stakeholders	5
4. Investigation Report	5
4.1 Students: Who Studies Engineering?	5
4.1.1 Demographics	5
4.1.2 Academic Backgrounds	8
4.2 Challenges in Learning and Teaching	10
4.3 New Methods of Learning and Teaching	12
4.3.1 Increased student support	13
4.3.2 Group learning	16
4.3.3 Software and Online Learning	19
4.3.4 Developmental Mathematics Programs	25
4.3.5 Other Learning and Teaching Methods	27
4.4 Methods of Assessment	28
4.5 Subject Content and Organisation	30
4.6 Oversight	34
4.7 International Site Visits	38
4.7.1 United Kingdom	38
4.7.2 USA	39
4.8 Some Overseas Comparisons	41
5. The Current State of Engineering Mathematics	42
6. Vision Statement	45
7. Recommendations	46
7.1 Recommendation 1: assumed knowledge	46
7.2 Recommendation 2: designated quantitative stream	47
7.3 Recommendation 3: statistics and stochastic modelling	47
7.4 Recommendation 4: joint mathematics curriculum committees	48
7.5 Recommendation 5: collaborative teaching	48
7.6 Recommendation 6: engineering mathematics staff expertise	48
7.7 Recommendation 7: on-line formative assessment	49
7.8 Recommendation 8: collaborative item bank	49
7.9 Recommendation 9: student help centres	49
7.10 Recommendation 10: boosting senior secondary school mathematics	50
8. Linkages	51
9. Further Development	51
Bibliography	52
Appendix 1: Mathematical Content for Engineers at International Universities Visited	i
Appendix 2: Literature Review	vii
Appendix 3: Questionnaire	lv
Appendix 4: National Symposium on Mathematics Education for 21st Century Engineering Students	lxi

Mathematics for 21st Century Engineering Students

1. Initiative Description

The landscape of Australian engineering mathematics education has been changing markedly. In response to new demands of the engineering profession and a diversifying student intake, many universities have independently been redesigning their curriculum, pedagogical strategy and assessment methods in mathematics subjects for engineers. Most Australian universities' mathematics and engineering departments face similar challenges of meeting the changing demands of engineering mathematics education but they tend to work independently to develop individual strategies. This scoping project includes a timely national review of mathematics teaching and learning strategies for engineering students. It is hoped that by reporting on many initiatives developed in isolation at individual universities, we will encourage a more open dialogue, better informing educators so that they may identify good practice. This will initiate a more concerted nationwide effort to improve education practice in this critical subject.

The Carrick Institute for Learning and Teaching in Higher Education Discipline Based Initiatives Scheme (DBI) has funded a scoping project to examine mathematics education for 21st century engineering students. The project has been coordinated by the Australian Mathematical Sciences Institute (AMSI). Having 27 member universities as well as support facilities including library access provided by the University of Melbourne, AMSI is ideally placed for such an exercise. In addition, the project team has made use of a national network of 12 Access Grid Rooms originally set up with the support of the Australian Centre of Excellence for Education in Mathematics, managed by AMSI.

To inform this report, we have completed

- an extensive literature review of mathematics learning and teaching practice for engineers,
- analysis of a questionnaire survey on student demographics, curriculum content, education practices and organisation, answered by the great majority of engineering degree-awarding institutions,
- site visits to a large number of engineering and mathematics departments in Australia plus a small number in UK and USA,
- a one-day workshop in which enthusiastic teachers of engineering mathematics were able to explain their approaches, connected to many participants through 16 access grid rooms distributed around the country and internationally, and
- many informative discussions with interested educators from the disciplines of engineering and mathematics.

Background

We felt it necessary to conduct this project because experiences in some of our member universities had led us to some prior conceptions of serious challenges in engineering mathematics education, yet we didn't have the full picture of national trends, norms and variability of current practices.

Engineering, like many other professions, has evolved significantly in the past couple of decades and is constantly changing to meet the needs of society. The engineering curriculum is adapting to provide students with an adequate foundation to enter the profession. Over the same period the student body has also changed significantly with an increase in the overall proportion of the population attending university, an increase in the number of international students and an increasing number of students in paid employment (Dobson, 2007). Engineering students need mathematical education but their backgrounds, abilities and attitudes vary widely. Mathematics and statistics educators are attempting to engage an increasingly diverse student body.

Engineering is perhaps the most important profession for the mathematics discipline. New developments in engineering have stimulated fruitful areas of mathematics research (constrained dynamics, control

theory, signal processing and queuing theory are a few examples). Engineering students are the single largest client group of university mathematics departments. The welfare of many mathematics departments depends on their ability to understand and respond to the needs of engineering education. However, there are widely varying opinions on the adequacy of past and present curricular and education practices in engineering mathematics.

The Carrick Discipline-Based Initiative on Engineering Education, run concurrently in 2007, broadened its purview to cover Supply and Quality of Engineering Graduates for the New Century (Australian Council of Engineering Deans, 2008). Late in 2006, there was released a report of the national review of mathematical sciences in Australia (National Committee for the Mathematical Sciences of the Australian Academy of Science, 2006). These two reviews already provide much information on the broader context of mathematics and engineering education in Australia. This allows us to concentrate more on the important issues of learning and teaching practices.

This project may prove to be important for mathematics service teaching even outside the client group of engineering. Mathematical developments lie at the heart of recent advances in biomedical science, commerce, and information technology. In this regard, engineering may be viewed as representative of a rapidly evolving technological discipline with a diversifying student body. Similar issues face mathematics educators when they work with students from many other disciplines.

2. Investigation Strategy

2.1 Advisory Committee

The Advisory Committee comprises approximately equal representation from both the engineering and mathematics disciplines. There are representatives from 15 Australian Universities (from all states), the Defence Science and Technology Organisation (DSTO), Engineers Australia (EA) and the Australian Council of Engineering Deans (ACED).

Karen Baker	University of Melbourne
Alan Bradley	Engineers Australia
Grant Cairns	La Trobe University
Pietro Cerone	Victoria University
Jim Denier	University of Adelaide
Gary Fitz-Gerald	RMIT University
Larry Forbes	University of Tasmania
Sam Fragomeni	Victoria University, ACED
Roger Hadgraft	University of Melbourne, AAEE
David Ivers	University of Sydney
Les Jennings	University of Western Australia
Robin King	University of Technology Sydney, ACED
Tim Langtry	University of Technology Sydney
Mark Nelson	University of Wollongong
Ian Porter	University of Wollongong
Jacqui Ramagge	Universities of Newcastle and Wollongong
Stephen Roberts	Australian National University
Tony Roberts	University of Queensland
Francis Rose	DSTO
John Simmons	University of Queensland
Patrick Tobin	Swinburne University of Technology
James Trevelyan	University of Western Australia
Lesley Ward	University of South Australia

We are grateful also to Archie Johnston (University of Technology Sydney, ACED) and David Panton (University of South Australia) who were active members of the advisory committee for most of its existence but were later diverted to other tasks.

2.2 Literature Review

As a first step to identify current practice in the teaching and learning of mathematics to engineering students, an extensive literature review was undertaken. Members of the Advisory Committee were requested to bring to light any unusual or outstanding practice. This involved searching the proceedings from engineering and mathematical education conferences as well as educational journals, following their leads to crucial cited documents. The review included international current practice.

A full literature review is available (Lopez, 2007). Here we summarise some salient points. The literature review found that many authors stress the importance of a solid mathematical education for engineers (Croft and Ward, 2001; Kent and Noss, 2000; Blockley and Woodman, 2003; Trevelyan, 2007). All mathematics is the ultimate form of logical rigour, helping to lay the foundation for good analytical and problem solving skills. While there is much debate about the content and amount of mathematics required for each engineering discipline there is little debate about the necessity of a foundation in mathematics. Kent and Noss (2002a, 2002b) have noted that some mathematical concepts have become so much embedded in engineering practice that engineers do not identify them as mathematics when using them in daily engineering practice, leading some to erroneously discount the importance of mathematics to professional engineering practice.

Most papers on engineering mathematics education could be classified according to four topics; problem-based learning (PBL), the multi-disciplinary approach, computer based methods (CBMs) and strategies that specifically address student variability. Nirmalakhandan *et al* (2007) claim that the more students participate in their learning, the greater are their learning achievements. Active learning is “learning by doing” rather than the traditional lecture format.

Advanced CBMs and PBL are most frequently claimed to be effective forms of active learning. Many authors reason that CBMs stimulate interest and enhance comprehension (e.g. Brenner *et al*, 2005, Mtenga and Spainhour, 2000, Naimark, 2002 and Colgan, 2000). Nirmalakhandan *et al* (2007) and Duran *et al* (2007) give empirical evidence. Morgenroth *et al* (2002) also argue that CBMs help students enter the profession, as most industries are now reliant on computers.

CBMs are often enhanced when used in conjunction with other innovative methods. They are argued to be particularly effective with a PBL approach (Duran *et al*, 2007). PBL is seen to be particularly effective in small groups; the PBL classroom environment mirrors many workplace situations (Smith *et al*, 2005). These findings do not however render traditional teaching methods redundant. Nirmalakhandan *et al* (2007) quantitatively show the effectiveness of integrating physical and computer models with the traditional theoretical model, showing that combinations of teaching methods help to tap into the different learning styles of students.

MATLAB is widely believed to be the most effective software in the teaching of mathematics to engineering students (Kent and Noss, 2000, Mtenga and Spainhour, 2000 and Waldvogel, 2006). It is seen to be particularly effective due to its ease of use (data is input as a simple list) and its wide use in the engineering profession. MATHCAD and POLYMATH are also highly regarded for similar reasons (Brenner *et al*, 2005 and Morgenroth *et al*, 2002).

Avitabile *et al* (2005) showed that lateral integration of learning materials across subjects may be taken further to a vertical integration model, where material used in earlier subjects is revisited later, interweaving learning experiences and concluding with a final year project in dynamical systems.

From the collection of literature, there emerges a view that the most effective engineering mathematics subjects are part of a well-designed engineering curriculum that enables students to understand the relevance and see the development of concepts over the entire course. It is crucial to address the increased student variability and to take into account differing learning styles. Easily accessible student support systems are also key to an effective engineering mathematics subject.

2.3 Site Visits

Helping Engineers Learn Mathematics (HELM): this project in the United Kingdom was identified as an example of good practice by the literature review. P. Broadbridge visited Loughborough University to speak with HELM directors, to visit the Mathematics Education Centre and to view their teaching aids.

Broadbridge also visited Duke University and the University of Delaware, which were also identified in the literature review as interesting centres of activity in engineering mathematics education.

A number of Australian institutions were also visited by our team members, sometimes in conjunction with site visits by the committee for the other parallel DBI project on engineering education, and sometimes as follow-up to our questionnaire responses. These institutions included; the University of Western Australia, Murdoch University, Curtin University of Technology, University of Technology Sydney, the University of Wollongong, the University of Melbourne, RMIT University, Victoria University and La Trobe University. The findings will be synthesised with the responses to the questionnaire, below

2.4 Questionnaire

A questionnaire was designed to explore the themes identified by the Advisory Committee, partly guided by the literature review. The questionnaire was designed in a qualitative format in an attempt to prevent the conveyance of any preconceptions of common practice and in the hope of allowing institutions to highlight any interesting practices currently in use.

Thirty-two universities were identified as offering Bachelor of Engineering degree programs. Appropriate representatives from both the mathematics staff and the engineering staff were identified and requested to complete the questionnaire or to redirect it to an appropriate nominee. The questionnaire was sent to both the Engineering Unit and the Mathematics Unit at the universities. 27 universities responded, including 14 who provided responses from both disciplines.

While we requested discussion of more advanced mathematical subjects traditionally taught by engineering staff, such as Signal Processing and Control Systems, the majority of respondents referred only to the subjects taught by mathematics staff.

2.5 National Symposium

A National Symposium on Mathematics Education for 21st Century Engineering Students (www.amsi.org.au/Carrick_seminar.php) was held on 7th December 2007, in conjunction with the Annual Conference of the Australian Association of Engineering Education (AAEE). The symposium took place at RMIT Access Grid Room (AGR), linked to several external groups in other states and internationally.

2.6 Terminology

Different institutions use different terms. To avoid confusion on administrative entities and components of instruction, terms used in this report have the following meaning:

Mathematics/Statistics Department: an academic unit whose chief responsibility is the teaching of mathematics and/or statistics. This may be a team within a school or faculty.

Engineering Department: an academic unit whose chief responsibility is the teaching of one or more specialisations of engineering. This may be a team within a school or faculty.

Subject: a study of a particular set of topics usually over a period of 12 to 14 weeks which is assessed as an individual element within a degree program

Mathematics Subject: a subject that contains at least 50% mathematics or statistics content.

Engineering Degree Program: the complete 3 to 4 year study program majoring in any type of engineering discipline or specialisation.

Engineering Students: students undertaking an undergraduate engineering degree program

Class: any period of time in which students are taught in timetabled groups.

3. Stakeholders

The peak representative body of senior engineering academics is The Australian Council of Engineering Deans (ACED). That body was supportive of our project from the outset, even hosting our first steering committee meeting prior to submitting the proposal. Our steering committee has always included two or three present or past Deans of Engineering. ACED was simultaneously funded by the DBI program to conduct its own separate project on engineering education. Broadbridge was a member of the steering committee for that project, taking part in some of our joint site visits to Engineering Faculties. Its project director was a member of our steering committee. We are grateful for having pre-release access to the ACED-commissioned report, “Addressing the Supply and Quality of Engineering Graduates for the New Century”.

The main professional association to promote engineering education is the Australasian Association for Engineering Education (AAEE). The President of AAEE was an active member of our steering committee.

The most prominent professional body is Engineers Australia. This body was represented on our advisory committee by its Associate Director (Accreditation). The project leadership team included two other engineering professors and a chief materials scientist based at The Defence Science and Technology Organisation (DSTO).

Our steering and advisory committees included several representatives of university mathematics departments. All states were represented.

The effectiveness of any education program can be gauged by the learning outcomes and degree of satisfaction of students. The study made use of student surveys on course effectiveness that had already been administered by the departments. On some joint site visits, arrangements were made to meet with student focus groups.

4. Investigation Report

4.1 Students: Who Studies Engineering?

There have been many changes to the student body over the past 20 years, with an increase in the overall number of students attending university, an increase in international students, an increase in the number of women studying at university and an increase in the number of students working while studying. This is coupled with an overall change in expectations of students. In an environment of centrally allocated quotas for government funded students places, increasing Higher Education Contribution Scheme (HECS) fees and widening participation, universities are competing to attract students who are becoming discerning consumers, now expecting the university as service provider, to fully prepare them for their chosen profession.

Widening participation and the need for more students to fill more places has led, in many cases to the lowering of entry prerequisites for courses. The increase in number of international students and increasing mobility of students between states has led to an increase in the variation of academic backgrounds of students entering each university. There have also been changes to Australian secondary school curriculum that have impacted on the assumed knowledge on entry into first year subjects.

4.1.1 Demographics

When asked about the changing demographics of engineering students over the past 10 years, many questionnaire respondents identified an increase in international students (15/27 institutions). Respondents also identified a greater number of students entering via non-traditional pathways. A couple of respondents identified an increased number of students in paid employment and an increase in female students. Many identified an increase in the number of students with lower mathematical preparation or ability. Comment was also made about the changing expectations of students and the move to the student as consumer, taking a more active interest in ensuring that expectations are met. Eleven respondents stated that they had noticed no change.

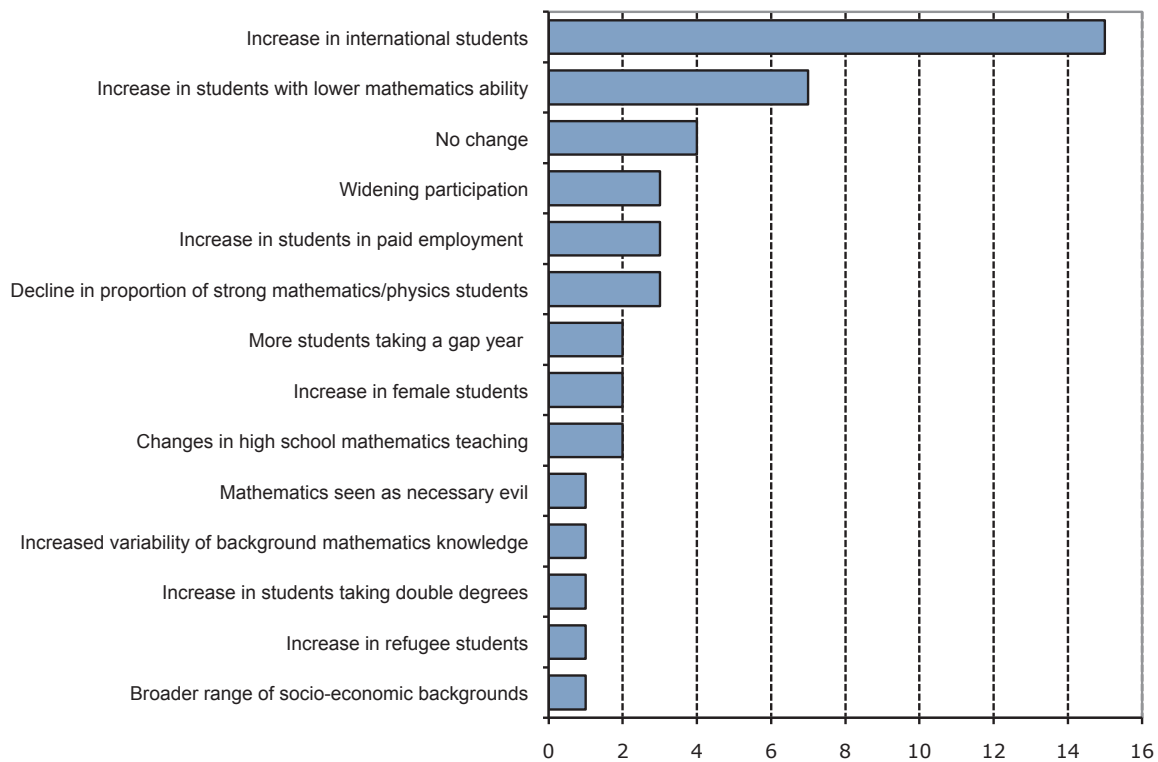


Figure 1: Categorised Questionnaire responses to: 3.2 Have you noticed any particular changes in the demographics of engineering students in the past 10 years?

International students

Total higher education student enrolments have increased by almost 17% from 2001 to 2006. However, the increase in the number of domestic students has been relatively modest at approximately 7%, while the number of international students increased by almost 60% in the same period. From 2001 the proportion of international students studying at Australian universities increased from approximately 19% to approximately 25%. (Department of Education Science and Training, 2007).

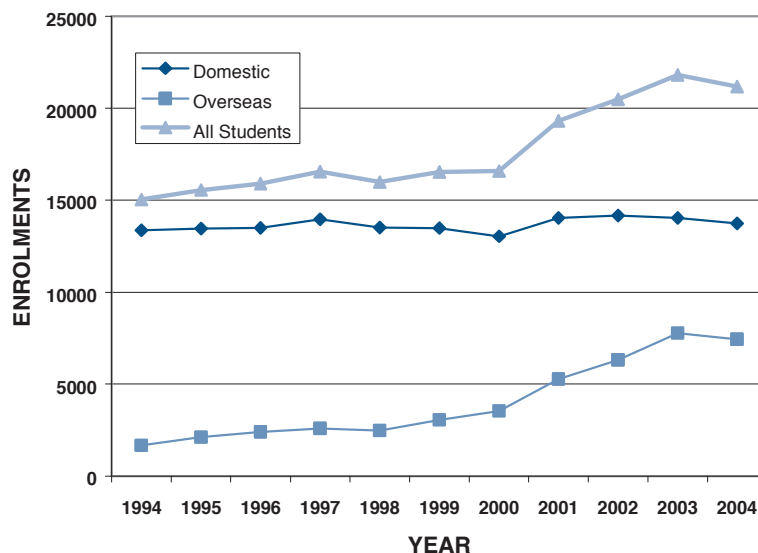


Figure 2: Commencing Engineering Enrolments (Engineers Australia, 2006a)

This large increase in international students has also been seen in engineering degree programs. Although the number of students commencing engineering degree programs increased by just over 40% from 1994 to 2004, the number of domestic students studying engineering remained stagnant with only a slight increase in numbers. In the same period the number of international students commencing

engineering degree programs more than quadrupled. Figure 2 shows that in 1994 the proportion of international students commencing engineering degree programs was approximately 11%. By 2004 this had risen to just over 35% (Engineers Australia, 2006a).

Gender

In the past 30 years the number of women attending university and studying science has increased rapidly. Overall, women now outnumber men in Australian higher education. The proportion of all female students has remained at around 55% since 1996 (Australian Vice-Chancellor's Committee, 2005).

While this greater proportion of women is also seen in science degree programs, women are still in the minority in engineering degree programs. The new enrolment of women between 1994 and 2004 in engineering degree programs has fluctuated around 14%, peaking at 15.7% in 2001.

This lack of female representation may also be seen when looking at numbers of women in the engineering profession. Estimates by Engineers Australia (2006a) using ABS Census data show that women make up less than 10% of the engineering profession.

Type of Attendance

Overall student attendance hasn't changed dramatically in the past 10 years. The proportion of students attending full-time has fluctuated around 59% with a peak of 63% in 2001. The proportion of students attending part-time has reduced from nearly 28% in 1996 to just over 21% in 2003. In the same period the proportion of students attending externally has increased from just over 13% to 15% and the proportion of those enrolled by multi-modal attendance had increased to just under 5% in 2003 (Australian Vice-Chancellor's Committee, 2005).

However, there appears to be a degree of discipline-specific variation in type of attendance, for example, science students are more likely to attend internally and full-time than students overall. In 2005 nearly 89% of natural and physical science students attended internally and nearly 80% of natural and physical science students attend full time (Dobson, 2007). Statistics could not be sourced specifically for engineering students.

Paid Employment

In 2006 three-quarters of all Australian students were undertaking some form of paid work throughout the whole semester. There has been a slight drop in the proportion of undergraduate students in this category. In 2006, 70.6% of full-time undergraduate students reported that they were in paid employment, compared to 72.5% reported in 2000. Likewise with part-time undergraduate students, 79.6% reported undertaking paid work in 2006, compared to 87.2% in 2000. However the proportion of full-time undergraduates in employment at some stage during the year rose from 78.1% in 2000 to 85.1% in 2006. Overall the number of undergraduate students in paid employment rose from 80.6% in 2000 to 85.5% in 2006 (James *et al*, 2007).

In 2006 full-time undergraduate students were most likely to be in casual employment (71.2%), but 5.3% reported working full time, while studying full-time. The majority of part-time undergraduate students were in full-time employment (57.0%). The majority of undergraduate students work at one workplace each week, but it should be noted that 17.3% of full-time undergraduates and 11.3% of part-time undergraduates reported working in two work places each week, while 2.7% of full-time undergraduates reported working at three or more workplaces per week (James *et al*, 2007).

The mean number of hours worked per week during semester for full-time undergraduate students in 2006 was 14.8 hours (with a median of 13 hours), while the mean number of hours worked per semester for part-time undergraduate students was 32.7 hours (with a median of 38 hours) (James *et al*, 2007). When asked about the impact of paid employment on their studies, many students reported an adverse effect. 40.2% of full-time undergraduate students and 55.3% of part-time undergraduate students reported *my work commitments adversely affect my performance in university* (James *et al*, 2007).

Changing Expectations of Students

Over the past 50 years there has been a change in the proportion of the population attending university and in students' expectations as to what they will gain from their university education. In the 1960s only about 5% of school leavers went to university. Typically these students did not pay fees, did not work to support themselves and studied full time. This proportion has steadily increased since the 1960s. The reasons students choose to enter tertiary study also appear to have changed. Generation X (born 1960-1984) saw an increasing emphasis on personal wealth and achievement, and seemingly there is a growing number of students who study to further their career, rather than for pure intellectual enjoyment (McCrindle, 2007). With the introduction of HECS in 1989 Generation X students have typically accrued large HECS debts.

Generation Y (born 1984-1995) school leavers entering tertiary education do so with 30-40% of their peers, pay HECS, over 70% of students work during semester, and have the option of flexible study, meaning degree programs can take a number of years to complete. This is coupled with increased diversity of students, with female students, and an increase in full fee paying international students. Changes to university financing have also led to competition between universities for students.

These changes to tertiary education have put students in a position to demand more from their university education (Department of Education Science and Training, 2002). Anecdotally it has been commented that students acting as "consumers" demand accessible course material, want to know the relevance of all they are taught and expect to be job ready on leaving university.

4.1.2 Academic Backgrounds

In response to the question, *"Have there been changes to the academic backgrounds of students entering engineering degree programs?"*, most universities stated a decline in mathematical preparation. However three universities stated that no dramatic changes had been noticed.

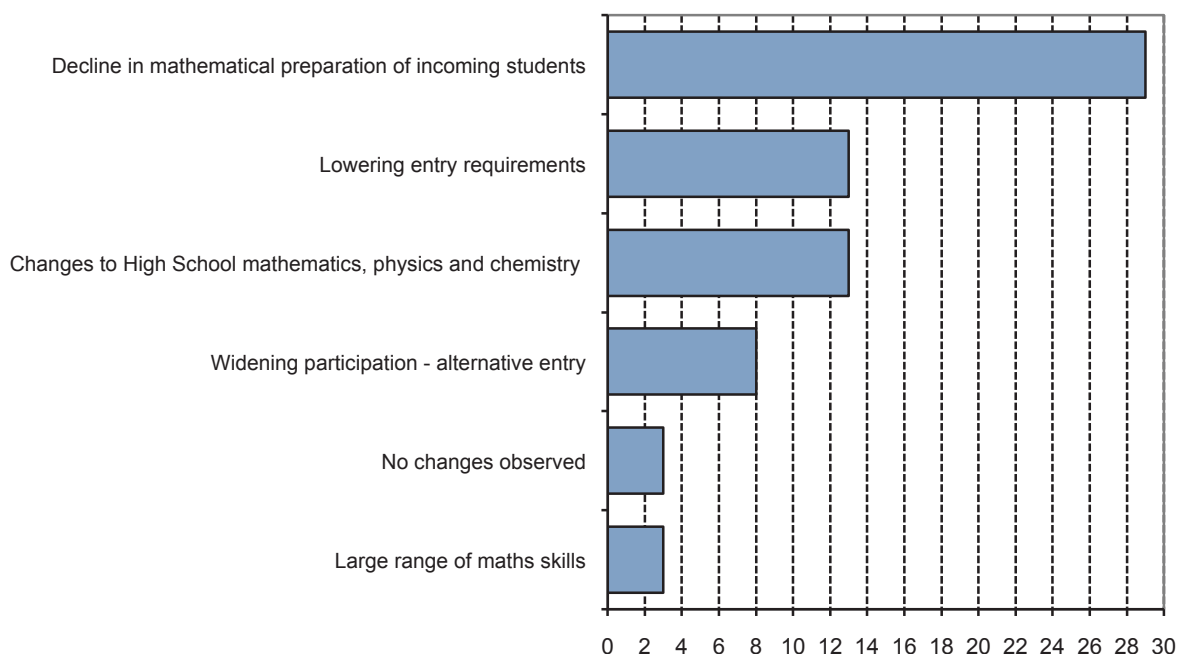


Figure 3: Categorised Questionnaire responses to: 3.1 Have there been any changes to the academic backgrounds of students entering engineering degree programs, including the effects of changes to high school mathematics and widening university participation?

Many respondents to the questionnaire attributed the decline in mathematical ability to a lowering of entry standards to engineering degree programs; the majority of universities have removed the higher level secondary school mathematics prerequisite. Many universities also now offer alternative entry routes through TAFE and Foundation Year studies; these students were also identified as less mathematically prepared or able in general. Concern was also expressed by a number of respondents about changes

to the secondary school mathematics and science syllabus and differences in syllabus between states, with particular concern about the standard of intermediate mathematics from secondary education in Queensland. Worryingly these concerns are supported by the findings of Barrington and Brown (2005), Barrington (2006), Barry and Chapman (2007), Belward *et al* (2005), Mullamphy *et al* (2007) and Engineers Australia (2007).

One respondent felt that mathematically and scientifically well prepared students were still coming from secondary school, but fewer are attracted to science or engineering programs. Further, he expressed a need for this problem to be addressed.

School Mathematics

A Senate Committee Report (2007) found: “There are some serious concerns about mathematics curriculum and syllabus standards in some states. It appears on the basis of evidence available that standards are declining in this subject, compared to other subjects, including English. The problems are at both the bottom of the school and the top: the failure to instil the required level of ‘numeracy’ in the primary school years; and the failure to encourage the required degree of rigour in a larger proportion of students in the senior secondary years.” Wilson and MacGillivray (2007) found that some students, with a senior algebra and calculus-based background need help with some basic mathematics, particularly in unfamiliar or multi-step situations. This further reiterates the importance of pre-senior mathematics to the consolidation of mathematical knowledge in senior years.

On top of the (perceived) decline in standards of school mathematics, Barrington and Brown (2005) found enormous variation in the Year 12 mathematics subjects between the states in Australia. Subjects vary in “their philosophy, in the mathematics covered, in the use of graphics calculators, and in their assessment. The differences are so great that no two states’ intermediate mathematics subjects could be described as equivalent” (Barrington and Brown 2005). As tertiary first year mathematics courses usually assume the secondary school curriculum knowledge of the state in which they are based, students moving between states will find a mismatch between the mathematics they have learnt at secondary school and the assumed knowledge for their first year mathematics subjects.

Year 12 Enrolments in Engineering Enabling Subjects

Students wishing to enter an engineering degree program generally require Higher School Certificate, or equivalent, English, Mathematics, Physics and Chemistry. Engineers Australia (2006a) presented the following data collected by the Department of Education Science and Training:

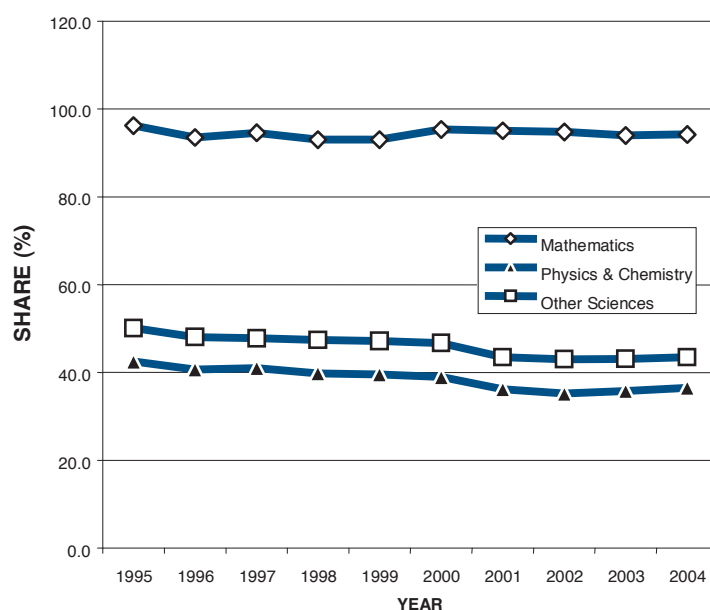


Figure 4: Shares of Year 12 Students Studying Engineering Enabling Subjects (Engineers Australia, 2006a)

The base line for these proportions has been taken as the number of students studying English. It may be seen that the trend in physics and chemistry enrolments is a concern for future enrolments in engineering degree programs. While the figures on mathematics enrolments may appear encouraging, it is important to remember that the level of study within the discipline is not evident. Barrington (2006) shows that counting enrolments can be misleading for two main reasons. Firstly, students enrolled in advanced, intermediate and elementary subject categories are not mutually exclusive. Secondly, procedures for enrolling students in mathematics subjects vary from state to state and over time, which can also lead to double counting of enrolments. These mathematics enrolments are considered in more detail below.

Participation in Year 12 Mathematics

While overall mathematics enrolments for Year 12 students appear to be healthy and to have increased slightly between 1995 and 2004, Barrington's (2006) statistics, displayed in Figure 5, show the total percentage of Year 12 students taking mathematics subjects, broken down to show the percentages of these taking advanced, intermediate and elementary mathematics subjects.

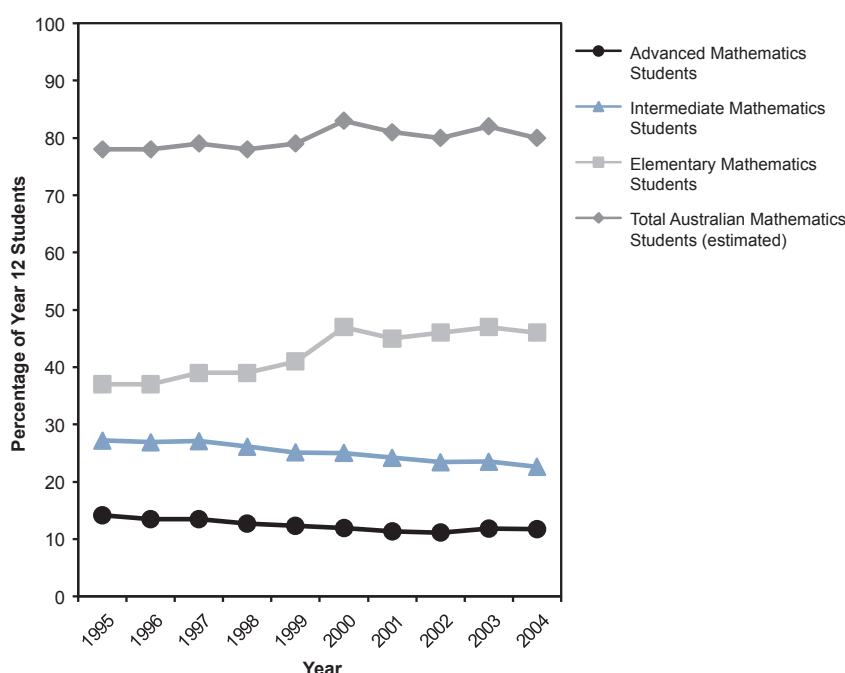


Figure 5: Year 12 Mathematics Students as Percentages of Year 12 Students (Barrington, 2006)

It may be seen from the graph that the majority of students are taking elementary mathematics subjects and the number of students taking intermediate and advanced mathematics subjects is steadily declining.

The National Committee for the Mathematical Sciences of the Australian Academy of Science (2006) further highlights the decline in school mathematics through international comparison. Although Australian school students achieve better than the international average in mathematics subjects at school, the spread across achievement levels is too wide. A comparison of Australian Year 8 students against averages of the top five countries shows that there is "extensive underachievement and small numbers reaching advanced levels" (National Committee for the Mathematical Sciences of the Australian Academy of Science, 2006). This poor preparation in lower years affects the number of students capable of studying intermediate and advanced level mathematics subjects in Year 12.

4.2 Challenges in Learning and Teaching

Lopez (2007) found a large body of research highlighting the need for educators of engineers to adapt to the changing nature of both the engineering profession and the student population in the 21st Century. A more diversified student population requires a more comprehensive learning support system. Changing industry requirements have led to the inclusion of management and communication subjects

in the engineering curriculum. There is much debate as to how these changes should be addressed. Institutions worldwide and within Australia are developing and implementing different adaptations.

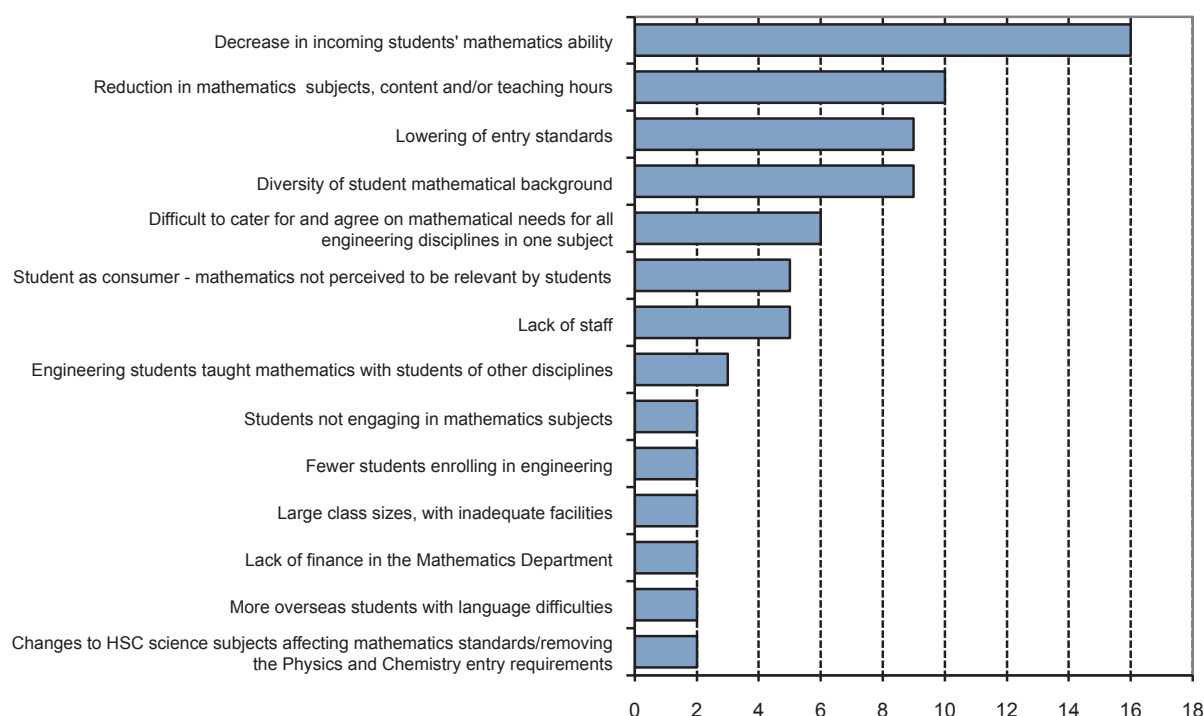


Figure 6: Categorised Questionnaire responses to 2.3 What challenges has your department faced in the teaching and learning of mathematics and statistics to engineering students in the last 5 years?

Most responding universities identified a decline in mathematical ability of entering engineering students, with a widening gap between skills and knowledge acquired at secondary school and assumed knowledge on entry to first year engineering mathematics. Many respondents cited this decline in mathematical ability as due to the lowering of entry standards – most universities have now dropped the higher level mathematics prerequisite which was previously standard for entry to engineering degree programs. The lowering of entry standards and increased number of international students has also led to the increased diversity of students' mathematical backgrounds. One respondent from Queensland felt that there is an increased gap between the skills of those students with Intermediate Mathematics and those with Advanced Mathematics. Observing that students with both Intermediate and Advanced Mathematics appear to be less affected by the (perceived) decline in emphasis on mathematics skills in upper primary and lower secondary, believing that students entering with good results in these subjects are well prepared for first year mathematics. Other respondents discussed the reduction of mathematical content in the secondary school science curriculum and the effect of this on the reinforcing of students' mathematical knowledge through other subjects. Several anecdotes recalled university students being surprised at the level of mathematics required for physics and chemistry.

Respondents spoke of the difficulty of attempting to cater for the mathematical needs for all engineering disciplines in one subject and often the difficulty of reaching a shared understanding between the mathematics and engineering departments about what is to be included in the curriculum. This seems to particularly be the case in institutions where engineering is taught in a problem or project based learning environment. A couple of respondents indicated that all science and engineering students are taught together in first and sometimes second year and spoke of the difficulty of delivering coherent, relevant mathematics subjects to these students. Comment was also made about the difficulty of engaging the increasingly diverse student cohort and the changing expectations of students with the move to the student as consumer, taking a more active interest in ensuring that expectations are met and expecting to know immediately the relevance of everything that is taught. Compounding this, respondents also spoke of a reduction of mathematics subjects for engineering students to make way for new management and communication subjects, but with the engineering department expecting students

to reach the same standard. The difficulty of teaching large classes with inadequate facilities was also mentioned by a couple of respondents. A challenge also identified is the lack of mathematics and statistics staff. One institution also spoke of the perceived threat of mathematics subjects being taken over by the engineering department.

4.3
New Methods of Learning and Teaching

Many questionnaire respondents indicated that they felt that there was overlap between question 3.3 “How has your department adapted to these [academic and demographic backgrounds of students] changes?” and 2.4 “Have you introduced any new methods of teaching and learning?” as many of the new methods of teaching and learning have been introduced in response to the changing backgrounds of students. To avoid repetition the responses to question 3.3 relevant to teaching and learning methods will also be discussed in this section.

Figures 7 and 8 show how responding universities answered questions 2.4 and 3.3. The black area shows those universities that identified the learning and teaching style in their answers to both questions 2.4 and 3.3, while the blue and grey areas indicate the number of universities that only identified the learning and teaching style in either question 2.4 or 3.3.

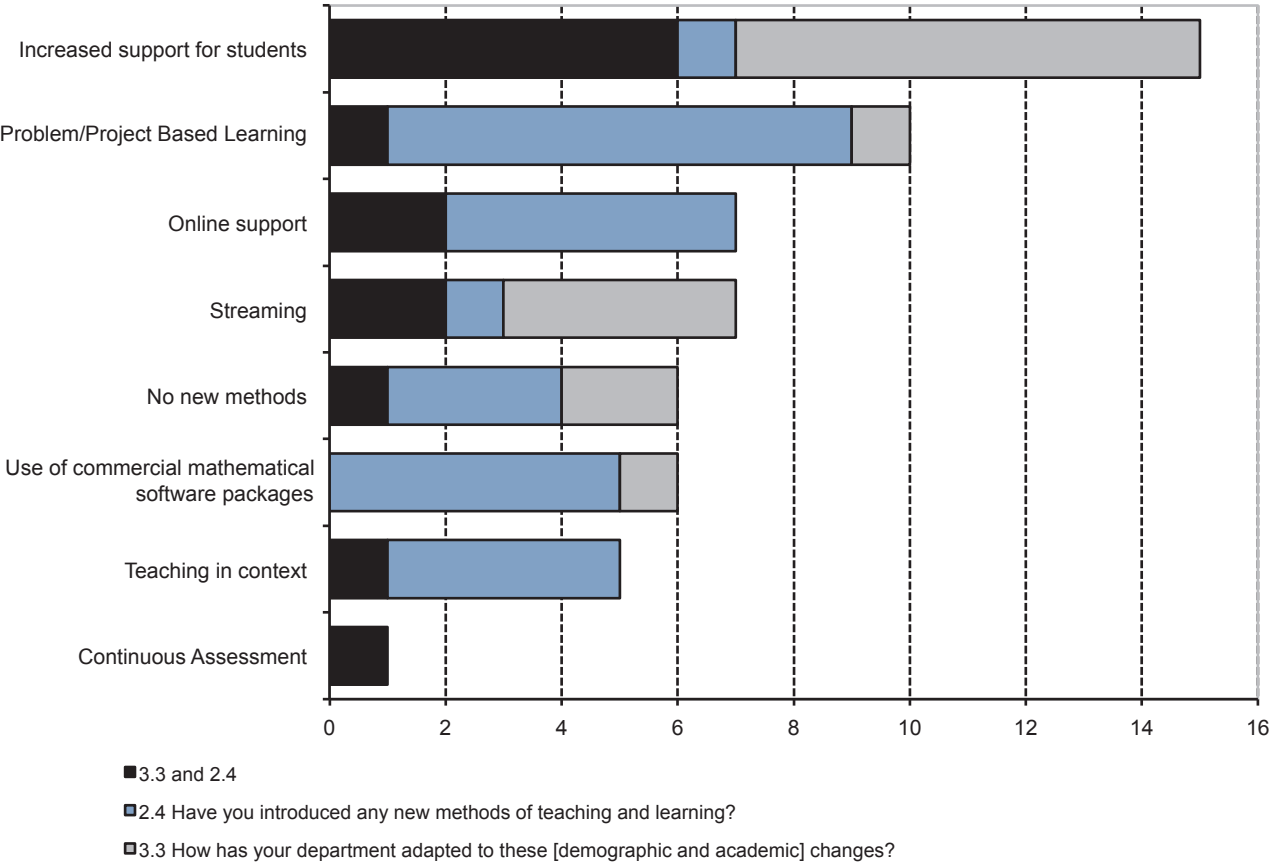


Figure 7: Categorised questionnaire learning and teaching styles discussed in responses to 2.4 Have you introduced any new methods of teaching and learning?and 3.3 How has your department adapted to these [academic and demographic] changes?

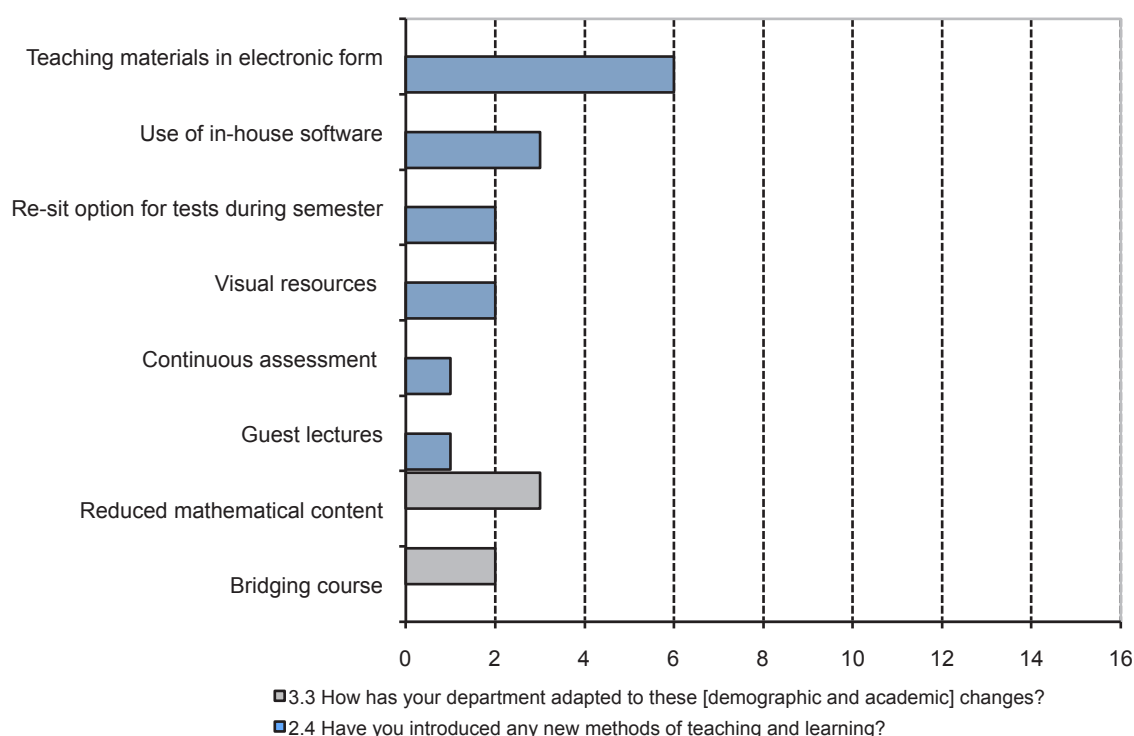


Figure 8: Categorized Questionnaire responses to 2.4 Have you introduced any new methods of teaching and learning? and 3.3 How has your department adapted to these [academic and demographic] changes?

4.3.1 Increased student support

Widening university participation, alternative entry routes, the increase in international students, changing expectations of students and changes to university funding have led to an increasing body of student focused research looking at attrition rates, learning styles of students and the effectiveness of student support. Internationally and in Australia there has been a significant extension of both academic and pastoral student support services offered in many universities.

Most institutions (15/27) identified an increase in the support offered to students. This increased support takes a variety of forms, with many institutions offering variations of peer assisted learning, often based on the supplemental instruction (SI) model developed in the USA. Some institutions operate mathematics learning centres, which are often manned during office hours, or for specified periods of time, by either staff or postgraduate students and can provide a variety of resources for students. At some institutions staff offer increased office hours to allow students to seek assistance. Extra tutorials are offered at some institutions, often being offered on a drop-in basis and for those students in need.

Additional tutorials and increased staff office hours were regularly mentioned by respondents as methods used to address the changing needs of the student body. This additional support is offered in a variety of ways, ranging from targeted and restricted to low-achieving students to voluntary attendance, available to all.

Supplemental Instruction

Many institutions now offer some form of SI based on a model originally developed and introduced in 1973 at the University of Missouri-Kansas City [<http://www.umkc.edu/cad/si/>]. SI is peer facilitated academic support for all students in units that have been identified as difficult by their high attrition rates. The sessions do not target high risk students, rather high risk subjects; sessions are available to all students and all can benefit from the program, avoiding the stigma of any 'remedial' label. SI usually consists of regular group sessions. The sessions are run by higher level students who have previously performed well in the particular subject. The sessions are informal, providing students with an opportunity to compare notes and work through problems. It has also been noted that the sessions

develop students' team working abilities, help students to take responsibility for their study and develop relationships with students in their courses (particularly relevant in Australia with so many students living off campus). Research has shown that the final examination marks of those students who attend the sessions on a regular basis are often significantly higher than those of non-attenders (Lewis, 2005). Attendance at SI sessions is voluntary, but highly recommended which can mean that "high risk" students (students the least likely to seek support) do not always benefit from SI.

A comprehensive meta-analysis of co-operative learning methods by Johnson *et al* (2000) consistently found that cooperative learning promotes higher achievement than competitive and individualistic learning, providing strong validation for its effectiveness. It has also been noted that higher level engineering students provide extra motivation to first and second year students in engineering mathematics as they are able to reiterate, from recent experience, the value of good mathematical foundations in more senior courses. While most Australian universities also employ some form of SI, it is not always offered for engineering mathematics subjects.

The University of Wollongong (UOW) operate Peer Assisted Study Sessions (PASS) [<http://www.uow.edu.au/student/services/pass/overview/index.html>] for MATH010: *Enabling Mathematics for Engineers* and MATH141: *Mathematics 1C Part 1*. O'Brien (2006) analysed the effectiveness of PASS for MATH141 students, finding that it is predominantly the less mathematically prepared students who attend PASS session and without adjusting for self-selection the effect of PASS attendance on final mark achieved is an average increase of 6 marks, and 10 marks after adjusting for self-selection. In student feedback obtained in autumn 2007, 100% of the 73 students surveyed in the final PASS session of semester agreed with the statement "Participating in PASS sessions has improved my understanding of subject content" (De Hosson, 2007).

The University of Western Sydney (UWS) also offer PASS for subject 300027.1 *Engineering Computing* (use of spreadsheets and implementing algorithms), compulsory for computer engineering students [<http://www.uws.edu.au/students/ods/lisu/pass>]. The University of Technology Sydney (UTS) offer SI in the form of UTS Peer Assisted Study Success (U:PASS) [<http://www.ssu.uts.edu.au/peerlearning/>] for *Mathematical Modelling 1* and *Mathematical Modelling 2*, both compulsory for the engineering degree program.

RMIT operates various forms of Peer Assisted Learning Schemes (PALS), which take the form of: Peer Assistance in Chemical Engineering (PACE) which was initiated by students and commenced in 2005 for particularly difficult parts of a foundational subject; 'Clinic', a mentoring program for first year students in the School of Aerospace, Mechanical and Manufacturing Engineering (SAMME) in semesters one and two; the School of Electrical and Computer Engineering offers a mentoring program for EET2246 *Introduction to Engineering Computation* (Algorithms and MATLAB programming); mentoring in this subject is to assist students who are technically focused and academically able to succeed in the shift towards independent learning. The mentoring program also aims to address social isolation, common in this student cohort.

At the University of Western Australia the Faculty of Engineering, Computing and Mathematics (ECM) offers one-on-one tutoring from trained senior Engineering undergraduates [<http://www.ecm.uwa.edu.au/for/students/1styreng/geng1004>]. The peer tutors are available (by pre-booking) for 45 minutes a week in ECM workshops for engineering mathematics subjects MATH 1010 *Calculus and Linear Algebra* and MATH1020 *Calculus, Statistics and Probability*.

Curtin University of Technology (CUT) offers learning enhancement clinics to first year students as part of the Engineering Foundation Year (EFY) [http://www.fac.eng.curtin.edu.au/EFY/id1_EFY%20Features/Student%20Support%20Ecfm]. Students may get academic assistance for all core units during business hours; tutors are either high achieving senior engineering students or staff members. Attendance is voluntary, informal and not monitored. Sessions are to help students who encounter a conceptual impasse and to engender self responsibility for study.

Mathematics Support Centres

Mathematics Support Centres are available for students at many institutions. These usually provide drop-in assistance for students at designated hours Monday to Friday and most offer students the opportunity to book one-on-one sessions with a tutor for areas in which they are really struggling. These sessions tend to be available to fill in gaps in students' assumed knowledge; they are not bridging sessions to tutor students who do not have the prerequisites for the subject.

The University of Adelaide operates the Maths Drop-In Centre [http://www.adelaide.edu.au/clpd/maths/drop_in/], which provides free assistance to all students studying mathematics and statistics. The Centre is available by appointment from 10:00 am to 4:00 pm, Monday to Friday during teaching weeks, but is also open at selected times during mid-semester breaks.

QUT offers assistance through the Maths Access Centre (MAC) [<https://olt.qut.edu.au/udf/QUTMAC/index.cfm?fa=dispHomePage&CFID=721&CFTOKEN=26278144>] to provide support and assistance to all students studying mathematics and statistics. There is a drop in room which is available to students for quiet study between 9:00 am and 6:00 pm. Academic staff and senior students are available in this room at various times, for a minimum of two hours each day. Up to three maths support sessions are also run weekly for engineering mathematics subjects (MAB180 and MAB182) to help students fill gaps in their knowledge. Support sessions are scheduled to ensure that they fit in with students' timetables. Exam preparation workshops are also held for first year engineering mathematics students. These sessions are held at key stages during the semester to help students develop study and problem solving skills. Exam preparation workshops usually consist of three 2-hour sessions in one day and are also student driven. Students may also download maths worksheets containing simple explanations, examples and exercises to complete in their own time.

Cuthburt and MacGillivray (2007) looked at the completion statistics for students commencing engineering degree programs in 2002, comparing those who made use of QUTMAC against those who did not. Clearly, as the sessions are voluntary students making use of QUTMAC are self selected - realising there are gaps in their knowledge and addressing these by attending the sessions. Obviously, at-risk students who attend the sessions are more likely to complete the course than those who did not recognise gaps in their knowledge and choose not to use available resources. Students who choose to make use of QUTMAC sessions have improved completion rates and are more likely as a cohort than the cohort as a whole, to complete the course without disruptions.

The Studio [http://fac.eng.curtin.edu.au/EFY/id1_EFY%20Features/Student%20Support%20Ecfm] is open for all first year engineering students at CUT. The Studio is laid out to mirror an open-plan office, providing a large communal area for study and small project meeting rooms that can be booked by students for private group study. Small project rooms are also used for 'Clinics', manned by tutors during office hours to assist students with any problems they may have. There are a number of hours of 'maths clinic' each week.

The Study and Learning Centre at RMIT [<http://www.rmit.edu.au/studyandlearningcentre>] offers drop-in sessions designed for students struggling with mathematics. Sessions are offered at various times Monday to Friday and are available to all students studying mathematics. The online Learning Lab [<http://www.dlsweb.rmit.edu.au/lisu/>] offers maths essentials in the form of online tutorials and hand-outs. Students are also able to get study skills advice and are encouraged to identify their learning style so they may plan how to study more effectively.

The UTS Mathematics Study Centre [<http://www.science.uts.edu.au/msc/index.html>] is manned between 10:00 am and 7:00 pm Monday to Friday by research students, lecturers and some student volunteers and is available on a drop-in basis for all students. UWS also offer semester workshops [<http://www.uws.edu.au/students/ods/lisu/mathssupport>] for *Mathematics for Engineers 1 & 2*. Workshops are voluntary, but students must register. Help with basic mathematics, calculus and statistics is offered to students through WebCT. Available resources include quizzes, booklets and useful web links.

ANU operates a mathematics drop-in centre for 8 hours each week, assistance is targeted at first year students and is available for two hours after each first year mathematics lecture. Staff believe that

students who attend the sessions gain real benefit and hope to direct more resources into increasing the service offered and encouraging more students to attend. It has been noticed by staff that it tends to be the more academically able and conscientious students who make use of the sessions and are looking at ways to encourage the less academically able students to take part. More resources are to be directed at the facility in 2008.

Tutorials

Many responding institutions referred to the introduction of extra classes and tutorials for engineering mathematics subjects. These take a variety of forms; remedial tutorials targeted at underachievers lacking the basic mathematical foundations, extra classes for the lower mathematics stream, an increased number of staff in tutorials, voluntary additional support tutorials and additional support tutorials for those lacking the pre-requisite knowledge.

A couple of institutions specifically referenced the transition from secondary school to university and a unified team response from both the mathematics and engineering departments. Staff in both departments are made aware of any concerns for particular students' performance or study characteristics and these are then addressed in a holistic fashion.

At UTS, first year engineering students studying *Mathematical Modelling 1* and *Physical Modelling* consistently stated that tutorials enhanced their learning experience (Wood, 2003), most requested more tutorials, in both time and number, and indicated that they would prefer the teaching balance to consist of more time in tutorials than in lectures. This preference was linked by many students to the difficulty of the transition from secondary school to university teaching. *Uncertainties and Risk in Engineering*, a second year mathematics subject taken by engineering students received very positive feedback for being a tutorial style course. Students expressed a number of difficulties with lectures such as, finding them difficult to follow, insufficiently varied in presentation, too fast paced, lecture sizes too large. Wood (2003) sees these problems more as students struggling with the change in teaching style than as problems with the lecturers themselves.

A model which was seen to be very effective at two universities in the past 20 years used an innovative system of small "whiteboard" tutorial classes in which several students, working in groups of two, had to simultaneously display their worked mathematics problem solutions on wall-to-wall whiteboards. In this format the tutor could stand in the middle of the room and immediately identify and assist those students that could be seen to be struggling. Unfortunately both have recently ceased that format because of the high staffing costs.

4.3.2 Group learning

Over the last couple of decades, in response to perceived industry pressure, there has been a growth in the inclusion of professional topics in the engineering curriculum, such as team-work, communication skills and report writing. Educators have explored many ways to incorporate these competencies without sacrificing technical content. Trevelyan's (<http://www.mech.uwa.edu.au/jpt/pes.html>) research on engineering practice found that engineers in the workplace do "lots of co-ordination in which technical knowledge is inextricably bound up with 'soft skills' and understanding of human behaviour" (Trevelyan, 2008), supporting the need for both the inclusion of management and communication skills and the acquisition of these skills in conjunction with theoretical learning throughout the engineering degree program.

Further supporting the inclusion of such competencies, in a survey of undergraduate engineering students at UTS, Wood (2003) found that students expressed a preference for real-life examples which put the mathematics learnt in lectures in context. Students also liked the problem-based approach, with plausible real-life examples, but they felt that long in-context examples spread over a few weeks of teaching were not sufficient and required additional smaller supporting examples to reinforce the principles in differing contexts. Wood (2003) found that engineering students were often unable to see the relevance of the mathematics taught, as in-context examples relevant to all engineering disciplines were not given.

There is also an increasing body of research on the benefits of active learning and its ability to cater to different learning styles (Dorfler, 2003; Mulligan and Kirkpatrick, 2000; Sazhin, 1998 and Vithal *et al*, 1995). These claims are quantitatively supported by Springer *et al*'s (1999) meta-analysis on the effectiveness of small group learning in science, mathematics, engineering and technology (SMET) in North American universities. Thirty-nine papers on small group learning were analysed. Academic achievement, persistence (retention) and attitude were measured. The study concluded that; “The main effect of small group learning on achievement, persistence, and attitudes among undergraduates in SMET was significant and positive” (Springer *et al*, 1999). In all three areas measured, students performed better when taught in small group learning situations.

Felder and Silverman (1988) found a mismatch between the learning styles of engineering students and the learning and consequently, teaching styles of engineering professors, identifying students as tending to be sensory, visual, inductive and active learners while engineering professors tend to teach in an intuitive, auditory, deductive and passive manner. He speculated that engineering students in general would prefer an active learning environment where students are involved in the learning process, suggesting such activities as group work, visual aids, open ended problems, etc. However additional sampling has led Felder (2002) to express caution, commenting that while active learning may address different learning styles he has found that students at university level only want to know what they need to learn for tests and exams, so therefore actually prefer the more traditional, prescriptive, passive approach of teaching. He does not however seem to consider the use of group work as an assessed component of the course.

Problem-based or Project-based Learning?

There is much confusion regarding the difference between Problem-based Learning (PBL) and Project-based Learning (PBLE), the terms often being used interchangeably. It is generally agreed that PBL is more directed to the *acquisition* of knowledge and PBLE to the *application* of knowledge. It is this definition that leads many to argue that PBL is not suitable for engineering degree programs, as much of mathematics, physics and engineering has a hierarchical knowledge structure, requiring topics to be learnt in a certain order to understand later concepts. Mills and Treagust (2003) contrast this with medical knowledge that has an “encyclopaedic structure”, with the order in which concepts are encountered less important and not affecting further learning.

It appears that in many instances in engineering education it is difficult to differentiate between PBL and PBLE as most subjects that call themselves problem- or project-based actually contain elements of both. Project-based work is widely used to allow students to practise the *application* of knowledge they have acquired in foundation subjects to real-world problems. However, when completing these projects students often have to do further research and *acquire* new knowledge to complete the project. It is therefore difficult to classify these subjects and approaches as either PBL or PBLE. We follow the lead of the reporting author or institution when we designate a particular practice as PBL or PBLE.

PBL/ PBLE is now widely used in Australian engineering degree programs. Lopez (2007) found many claims that PBL is particularly effective in engineering education, as small teams working on open-ended problems mimic the work situations of most engineers.

Victoria University now uses a PBL approach in all engineering subjects. Mathematics is taught to engineering students in small groups (20-30 students in a class); there are no large group lectures. Students have three hours interactive class time per week and one hour working on a group project that is an engineering problem related to the topics covered in the three hour interactive class time. Staff have found that students become very engaged while solving in-context engineering problems and have found that it is very effective in developing and reinforcing mathematical skills. One example of an in-context problem enjoyed by first year students involved the use of differential equations to model fluid flow in a mixing bowl. Mathematics department staff collaborate with engineering staff to develop such examples for students. PBL in first year engineering subjects has had positive feedback from students. Many comment favourably on the opportunity to develop team working skills, on peer assessment and on the social aspect of team work as a positive introduction to first year at university (Jayasuriya and Evans, 2007).

An interdisciplinary PBL approach is used to teach *Engineering Computational Methods* to first year engineering students at ADFA (Barry and Webb, 2006). The subject comprises six real-world engineering problems, developed in conjunction with the engineering department. Each topic is studied over a two week period, with two lectures and four hours of supervised computer laboratory sessions. When students are presented with the problem, they must then convert it to a mathematical formulation, design MATLAB code to analyse the problem and then reinterpret the answer as an engineering problem. Students must produce and submit a report as they would in the workplace. These six assignments account for 50% of students' final grade, with a closed book exam, consisting of small problems designed to reflect the knowledge gained from the assignments, for the remainder. While the logistics of developing this subject among three departments and three lecturers were difficult, once in place, lecturers have noticed many benefits of the course. Students learnt how to write well structured assignments, became comfortable with computer algebra software, developed good team working skills and gained skills in the overall approach to solving engineering problems.

A similar subject is also offered at the University of South Australia (Colgan, 2000). Teaching is divided into three hours of formal lectures, a one hour tutorial and one hour of computer laboratory. While the first three weeks of computer laboratory sessions act as an introduction to MATLAB, for the remainder of the semester students must complete two group projects. One project is a real-world engineering problem, the second is a report on a topic from the textbook not covered in class. Projects contribute 10% to the final grade. As with *Engineering Computation Methods* at ADFA, students develop report writing, team working, computational and self-study skills.

At CQU there has been an increase in "in-context" PBL for engineering students, with some engineering problems taking a week or longer to investigate and resolve. Project-based learning (PBLE) was also trialled at CQU in 2007. While it is widely believed to be effective, PBLE needs further development. At CUT it is believed that PBL and PBLE motivate and engage students, highlighting the relevance of topics being taught. Griffith University is also introducing discipline-specific PBL.

Close collaboration between mathematics and engineering staff at QUT facilitates problem-linked learning in mathematics, with the use of engineering problems in mathematics exercises. At QUT, staff believe that problem-immersed learning is not suitable for mathematics as staff believe it increases students' difficulty in transferring mathematics to new contexts. Britton *et al* (2005) investigated this, looking at the ability of science students to transfer mathematics and attempted to quantitatively measure this by creating an index, which was later modified by Roberts *et al* (2007). They found that while students who scored high marks in the in-context questions in the test were able to transfer mathematical knowledge to out-of-context mathematics questions, a large proportion of those who scored average marks in the in-context mathematics question were unable to answer the out-of-context questions. The findings of this research indicate that teaching mathematics in-context to less academically able students limits their ability to use the mathematics in any other context.

The Monadelphous Integrated Learning Centre (ILC) [<http://www.ecm.uwa.edu.au/about/ilc>] is to open in 2009 at The University of Western Australia (UWA). Several individuals and companies are generously contributing funds towards this initiative (for details see the above website). The design for this new Centre has been based on the ILCs at University of Colorado at Boulder [<http://itll.colorado.edu/ITLL/index.cfm?fuseaction=Home>] and Queen's University Live Building ILC [http://livebuilding.queensu.ca/contact_about], developed to accommodate a Project Based Learning Experience (PBLE) approach to teaching inspired by Aalborg University [<http://www.aau.dk>]. The ILC will mimic the industrial workplace, with facilities that integrate project management from design through development to prototype to be presented to the client. Students from different disciplines and different years will interact on projects, allowing students to benefit from peer mentoring, developing leadership skills and further simulating the work place environment with team members having differing areas of specialisation and varying years of experience. Students will also interact with industry professionals. In the design phase students will have to draw on the foundation theory and mathematics that they have learnt in first and second year in the core compulsory subjects, which include fundamental and advanced mathematics subjects. In the long

term UWA hopes to build a new Life Integrated Learning Centre modelled on the Queen's University Live Building ILC [http://livebuilding.queensu.ca/contact_about].

A first year mathematics subject for engineering students at the University of Queensland incorporates a group 'in-context' project in which students explore a relevant mathematical topic of their choice, produce a poster and give a short presentation. This allows students to develop an appreciation of the inter-disciplinary nature of mathematics and the development of communication, team-work and research skills. This small project has been noted by staff to increase students' interest in mathematics (Worsley *et al*, 2007).

Monash University has introduced ENG2091: *Advanced Engineering Mathematics A* (Donea and Lun, 2007) that is designed for chemical, civil, materials and mechanical engineering students. The subject aims to show how mathematical theory can be used in engineering practice, for example: animated gifs for curls and divs, example involving guitar strings played by a student and heat conduction. The use of in-context engineering examples in lectures and classes has been well received by students, with most students indicating via satisfaction questionnaires that the examples aid their understanding of the topics.

Most classes in most engineering mathematics subjects at RMIT spend significant time discussing 'in-context' examples relevant to students' discipline area to motivate the development of theory. Some engineering subjects are completely PBL in nature, all theory developed as a consequence of, and driven by, the need to address one or more engineering applications, while other subjects are more traditional in that they develop the theory first and then use the techniques and methods to solve relevant problems if time permits. Mathematics subjects are supported by practice classes and WebLearn activities, which provide students with both summative and formative feedback (Fernandez and Fitz-Gerald, 2004b).

Puzzle-Based Learning

A puzzle-based learning course is to be offered as a unit within *Introduction to Engineering* and as a full semester subject at the University of Adelaide in 2008 (Michalewicz and Michalewicz, 2007). Puzzle-based learning attempts to connect thinking and problem solving skills with mathematical awareness. PBL and PBLE usually deal with complex situations where there is not one clear way of proceeding. The emphasis with these approaches is how to deal with the complexity of the task and how to integrate a number of techniques, with students realising they need to learn some new knowledge before they can solve the problem. Puzzle-based learning may complement PBL and PBLE. Puzzles more often but not always have one correct answer and may appear to be deceptively simple. This contrasts with the traditional learning approach wherein students study a topic and then answer very structured questions before moving on to the next topic. Puzzle-based learning shows the importance of mathematics in solving the puzzles, challenging students to draw on previously acquired mathematical knowledge and highlighting the relevance of mathematics in all aspects of problem solving. Michalewicz and Michalewicz (2007) believe that puzzles allow students to understand how they have reached an answer and to understand how they may apply the same mathematical techniques to other real-world problems. Just as importantly, it has been found that many students find puzzle solving to be enjoyable and a motivation for learning mathematics.

An example of a puzzle:

The River Crossing Problem: A man has to take a wolf, a goat, and some cabbage across a river. His rowboat has enough room for the man plus either the wolf or the goat or the cabbage. If he takes the cabbage with him, the wolf will eat the goat. If he takes the wolf, the goat will eat the cabbage. Only when the man is present are the goat and the cabbage safe from their enemies. All the same, the man carries wolf, goat, and cabbage across the river. How has he done it?

4.3.3 Software and Online Learning

Rapid advances in computing in the past couple of decades, and the increased affordability of computer hardware has meant that the world wide web and software applications have become integrated into daily life. Computer technology is now commonplace in teaching and learning at all levels and is increasingly being incorporated into engineering mathematics subjects.

Since the introduction of computational techniques courses in the 1960's, there has been widespread debate about the use of computer technology in the teaching of mathematics in general and more specifically in the teaching of mathematics to engineering students. Many argue that students do not learn the “nuts and bolts” since the inbuilt computer code initially obviates the need for experience and conceptual understanding when the algorithms are first accessed. Others believe that computer technology has revolutionised the teaching of mathematics. In between the extremities of avoidance and compulsion, there is a strategy for the most effective use of software in the teaching of mathematics to engineering students, but research is needed to find it.

Most institutions now use some form of learning management system (LMS). WebCT and Blackboard were the two mentioned most often by educators responding to the questionnaire and at site visits. It appears that at most institutions lecturers are free to make as much or as little use of the LMS as they wish, meaning that there are great differences in the online presence of subjects. Some subjects will have all lecture notes, class notes and solutions to assignments available online (time released), with quizzes (not normally counting towards the final grade), a discussion blog in which the lecturer or tutors may take part, worked examples and on-line submission of assignments. Others will have no teaching and learning resources available to students online. Concern was expressed by one lecturer that if he put all teaching materials online, students would not attend lectures and tutorials. Increased accessibility of teaching and learning resources available to students at times convenient to them appears to be seen by many as one effective way of helping to address the changing needs of the student body. However no one has seriously claimed that this mode of dissemination could adequately replace interaction with an inspiring teacher.

Online quizzes through the LMS or in some instances via in-house software are increasingly being used in the teaching of engineering mathematics, with many respondents identifying the need for these online resources due to decreased numbers of mathematics staff and increased lecture sizes making it difficult for lecturers and tutors to provide detailed feedback for individual students and to quickly mark tutorial assignments. Online quizzes allow students to test their mathematical skills at times convenient to them with instantaneous feedback. Questions vary from multiple choice (for the basic LMS tests) to questions with many parts that must be entered as algebraic expressions (when in-house software has been developed). The detail of the feedback varies from binary correct/incorrect to targeted feedback on individual answers explaining why the answer is incorrect and suggested further reading on the topic if the student is still struggling.

Commercial Software

Supporting the findings of Lopez (2007), MATLAB is the most commonly used software package (20/27 responding universities). Excel was the second (10/27) and Minitab third (8/27). Surprisingly only 2/27 responding universities use MATHCAD and no responding universities use POLYMATH. The majority of universities use one or two packages (15/27), but ten universities use between three and five different software packages. Only two universities reported that no software packages are used in the teaching of mathematics to engineering students.

Some respondents stated that they found modelling software invaluable in the teaching of mathematics to engineering students as it has enabled them to better contextualise the teaching of engineering applications to students. Many institutions set group projects for students using some form of computer algebra system. The increased use of software packages in assignments is seen by some as a response to the perceived need for engineers to become proficient in using packages that are commonplace in the professional environment.

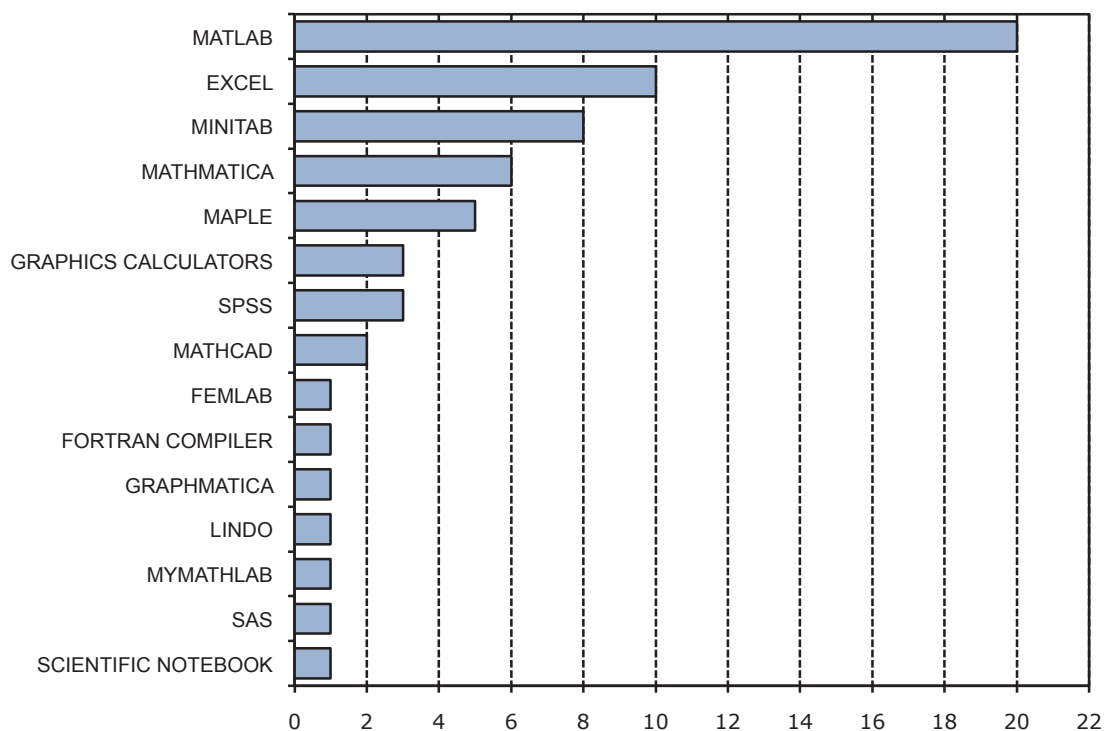


Figure 9: Categorical Questionnaire responses to 2.2 What mathematical and statistical packages do engineering students use in class?

The use of MATLAB is incorporated in the PBL subjects offered at the University of South Australia (Colgan, 2000) and the Australian Defence Force Academy (Barry and Webb, 2006). As there are few textbooks incorporating the use of MATLAB at first year level, both institutions have developed their own supporting resources for students (e.g. Barry and Davis, 2008). Both subjects combine active and passive learning in an attempt to further engage students and to highlight the relevance of the foundation material to the engineering degree program. Students must design MATLAB code to solve assessed engineering problems. The University of Wollongong has developed a series of online tutorials for students to introduce them to MATLAB (Baafi and Boyd, 2001).

Maple and Mathematica have been used in the teaching of mathematics at RMIT for over 15 years and are regularly used in engineering mathematics subjects (Fitz-Gerald and Healy, 1994). Blyth (2005, 2007a, 2007b) is an enthusiastic proponent of Maple in the teaching of mathematics at all levels. He ascribes to the computer algebra immersion mode of teaching and learning, whereby Maple is used for all class presentations, computer lab classes and assessment. Students are taught face-to-face in the computer laboratory. All Maple files for teaching, assignments and exams are downloaded from the internet by students. All student work is carried out, submitted and returned with feedback as Maple files via the internet.

Keady [<http://www.maths.uwa.edu.au/~keady/GKmplTeach.html>] is another enthusiastic supporter of computer algebra systems, personally favouring Maple in teaching mathematics, first introducing it to teaching and learning at UWA in 1989. At UWA, in the second year engineering mathematics subject MATH2040 students use both Maple and MATLAB [<http://www.maths.uwa.edu.au/~keady/Teaching/M2040/index.html>]. Keady has also developed a series of worksheets to assist in teaching a third year engineering mathematics subject with Mathematica [<http://www.maths.uwa.edu.au/~keady/Teaching/3M1/Assgts/index.html>].

To assist engineering students with learning a large number of mathematics topics in a short amount of time, concurrently with requiring the foundation knowledge in their engineering subjects. Bloom (2007) introduced the use of Scientific Notebook in the teaching of mathematics to engineers. Scientific Notebook allows students to practise and check their answers in their own time, without having to

spend a great deal of time learning a programming language a working knowledge can be learned in 20 minutes. Mathematics and text is easily entered using toolbars, and appears on the screen as it would be written. Bloom (2007) has also found that scientific notebook forces students to enter the expressions in the correct format, finding previously that hand written work is often written sloppily by students – particularly with Fourier series. There are obvious problems with solely relying on Scientific Notebook. Students are taught in weekly lectures, where scientific notebook is mentioned and weekly tutorials, where students answer questions both with scientific notebook and by hand. Students are assessed with assignments every two weeks where students must answer questions by hand, then check them with scientific notebook - students must hand in both solutions. An assessed mid-semester test is 50% Scientific Notebook and 50% by hand. The final examination must be answered totally by hand. There has been very positive feedback from students, finding that Scientific Notebook greatly aided their learning. Anderson *et al* (2000) are also proponents of the use of Scientific Notebook to assist the teaching of mathematics to engineering students.

Loch and Donovan (2006, 2007) trialled the use of tablet technology in the teaching of engineering mathematics subjects *Calculus and Linear Algebra 1 & 2* and *Discrete Mathematics* over three consecutive semesters. This technology was used in conjunction with workbooks, developed for all first year and some second year mathematics subjects at the University of Queensland, available for students to download in pdf format. The workbooks are to be used in lectures and have spaces for students to add comments and working. Graphics tablets were used for *Calculus and Linear Algebra 1 & 2* and a tablet PC used for *Discrete Mathematics*. The software was used to “write” on the projected pdf images during the lecture. There were various technical difficulties with one of the tablets used and the tablet PC. Student feedback indicated that the majority (80%) found the use of the tablet where there were no technical difficulties beneficial to the learning experience, but indicated a small preference for the use of over head projectors in the subjects where there were technical difficulties. However, students overwhelmingly (65-95%) stated that writing during lectures helps their understanding. Additional benefits with tablet technology include the ability to easily save and upload to the LMS annotated lecture notes, flexible lecture structure responsive to the needs of the students, the ability to backtrack to earlier sections, and allowing students to be actively involved in learning.

Excel spreadsheets have been used in the teaching of mathematics subjects for engineers at the University of South Australia to illustrate results. In a bridging program [<http://www.unisa.edu.au/study/progcourses/undgradpdf08/foundation.pdf>], which provides an alternative entry pathway for students to engineering degree programs, when examining the meaning of parameters of functions, students alter parameters in the spreadsheet and are able to visualise the effects of the alteration. Students then perform the tasks in MATLAB to familiarise themselves with this software. Other calculus topics using this method include; limits, Euler’s Method, Newton’s Method, logarithms and exponentials and Riemann Sums. A first year mathematics modelling subject uses spreadsheets to illustrate curve fitting using least squares methods and Markov modelling. Student evaluations have found that students consistently give good feedback about the practical sessions using Excel.

In-house Software

Some universities have developed in-house software to compensate for a decrease in mathematics and statistics staff, to reduce the marking burden of the remaining staff, and to allow students to study at times convenient to themselves. The use of software can also lead to more rapid diagnosis of basic misunderstandings than is possible in the lecture-assignment-tutorial cycle. Simple features such as randomisation of parameters and pattern matching can lower the risk of plagiarism. This software allows, or in some cases, forces students (with assessments to be completed before they can progress to the next stage) to practise mathematics, and it provides rapid diagnosis of, and feedback on, errors.

CalMaeth (Dynamical Teaching Solutions software interfaced with Mathematica) was developed at the University of Western Australia (Judd, 1996). Questions are randomly generated for individual students, usually differing only by variation of constants or transcendental functions, to keep questions at a

uniform level of difficulty. Questions are multi-part to reinforce the steps of solving complex problems and to allow the software to diagnose where student errors take place. An expression editor allows students to check that they have entered the answer they intended. If an incorrect answer is entered the software attempts to generate an appropriate diagnostic statement. Feedback starts with general statements about an error and then becomes progressively more specific. Each error also has a severity rating (10 for trivial arithmetic error – 50 for a serious misconception); this rating allows students to perceive the severity of a mistake. These ratings are also used for marking assessed questions – students may attempt the questions as many times as they wish, but lose marks in proportion to the severity of their errors. If CalMaeth is unable to determine what error the student has made it will respond with a long diagnostic, explaining what the student ought to be doing. The software keeps track of each student's progress, individual statistics may be viewed and summary statistics may be created – histograms and scatter plots of quantities such as average tutorial mark, average completion time, average number of attempts, exam marks and tertiary entrance scores. The software has had excellent feedback from both students and staff. A related system has been used at Macquarie University.

Assessment in Mathematics (AiM) [<http://maths.york.ac.uk/moodle/aiminfo/>] is a computer aided assessment system. The original version was developed at the University of Gent in Belgium, but has subsequently been significantly revised by various authors. AiM is mostly written in Maple programming language and is open-source. Answers are entered by students in Maple syntax; they may parse their answers to check they have entered the answer they intended, before submitting the answer for marking. AiM can provide comprehensive diagnostic feedback on incorrect answers. It can also be set up to show the student how their answer is incorrect, for example if a student solves a system of equations incorrectly, AiM can substitute the incorrect answers back into the equations and show that it does not work.

With the creation of Maple TA [<http://www.maplesoft.com/products/mapleta/>] and the subsequent licensing issues related to using Maple software with AiM (an open-source CAA), Sangwin (2006) has created System for Teaching and Assessment using a Computer algebra Kernel (STACK) [<http://www.stack.bham.ac.uk/>]. STACK uses the CAS Maxima [<http://maxima.sourceforge.net/>] which is open source. STACK operates in much the same way as CalMaeth and AiM, answers can be entered in algebraic form, an expression editor is provided for students to check their responses, diagnostic feedback is provided to students and they can be allowed a number of attempts at questions.

Weblearn interfaced with Maple (Fernandez and Fitz-Gerald, 2004a) is used at RMIT to enhance the learning experience for large group lectures (focusing on enabling Mathematics subjects in first year). Maple is used to provide targeted feedback to students when attempting practice questions, recognizing common errors and providing further explanation and suggested reading for more detailed explanation. Maple allows students to enter answers in a loose format and enables students to check that they have entered what they intended before submitting their answer - via an expression editor which shows the formulae entered in a more familiar written format.

Also at Swinburne University of Technology, to aid the laborious task of creating formative tests on the LMS Blackboard, Tobin and Lozanovski (2007) developed a modular software approach to the development of tests for students, semi-automating the process. Multiple-choice questions are generated with Mathematica and transferred to Microsoft Word, after which Respondus is used to upload the formatted multiple-choice questions to Blackboard. This approach means that for each question there is an associated canvas pool of hundreds of randomly generated, distinct versions of the question. In multiple-choice questions the number of distractors can be randomised as can the placement of the correct answer. All canvas pools can be easily updated and linked notes can be updated without affecting active tests. Student feedback for the online testing system has been very positive, with the majority of students finding multiple-choice tests a good way to monitor and check their progress.

MOMUS Tutor, coded in Macromedia Director, has been developed at Monash University to provide an online “tutor” for mechanical engineering students (Field *et al*, 2003). By animating machines and illustrating the need to isolate portions of the machine in some modelling processes, staff aimed to provide engineering students with realistic, practical examples of theory which have been found to

motivate engineering students. The menu pages provide students with an introduction to the software, describing the purpose and how it should be used. Students then view a video file, outlining the in-context problem. Specific text is authored for each problem, students are asked to construct a line diagram model for a part of the image under defined external conditions that may be used in the solution. The answer has several components. The tutor is programmed to diagnose the students' answers, offering appropriate feedback. Evaluations of MOMUS Tutor indicated that students who used it achieved higher on the examination problem related to machines.

At UTS, a MATLAB based visualisation package has been developed for the teaching of Optimisation Methods and Applications in Engineering to help students to understand the advanced mathematics (Lui, *et al*, 2007). The developers have found that students are usually more interested in and motivated by practical applications. The visualisation package consists of six modules associated with various optimisation algorithms. Practical optimal design problems from various disciplines are included. Student feedback on the package has been positive, with the majority of students commenting that they found the subject easy to follow. Cross disciplinary learning is encouraged in this subject with students encouraged to bring assignment problems from other subjects which may be solved using optimisation algorithms, further highlighting the context of the mathematics being taught.

A Coordinated Approach

It is clear that there are many innovative CAL and CAA systems being developed at various institutions, both nationally and internationally. In the majority of cases this software is being developed independently. It is clear from questionnaire responses that there is a great deal of overlap between the base software used at most Australian institutions, with 20 institutions stating that MATLAB is used in learning and teaching.

Keady *et al* (2006) argue that while CAA systems will come in and out of fashion, sharing systems, question databases, etc. is crucial for progress to be made. Institutions often develop very similar CAL or CAA systems independently, when there is the possibility of streamlining a small number of CAL or CAA systems.

Open-source systems such as STACK [<http://stack.bham.ac.uk/stack/>] have great potential for CAL and CAA as institutions may download these systems for free and then alter them as required – this obviously requires technical know-how, but removes the initial outlay for commercial CAL and CAA systems. It is crucial that improvements and changes are shared to ensure effective development.

It seems that a forum for information and question sharing would be of great benefit to Australian mathematics and engineering educators. Keady *et al* (2006) point out the economic benefits, in time spent developing the systems and particularly in time spent authoring questions.

Hadgraft (2007) is also a proponent of a coordinated approach to CAL and CAA, arguing that it has been poorly integrated into teaching at most institutions, believing that through online assessment students can develop skills at their own pace and test themselves to make sure they understand.

Question sharing may be particularly effective in mathematics education for engineering students, where many respondents to our questionnaire mentioned the difficulty of finding good engineering examples, at the correct level, to display to students the relevance of the mathematics being taught.

A project in the UK built a Remote Question Protocol (RQP) (<http://mantis.york.ac.uk/moodle/course/view.php?id=14>) which allows the use of appropriate item engines, enabling the sharing of questions, even when they are authored for different CAL and CAA systems.

4.3.4 Developmental Mathematics Programs

There is much debate as to the effectiveness of developmental mathematics programs on student retention and achievement. It is difficult to measure the effectiveness of these programs as there are usually many other factors influencing the success or failure of students entering into such programs. Lesik (2006) investigated the causal impact of developmental education programs on student retention by embedding a regression-discontinuity design within the framework provided by discrete-time survival analysis. She concluded that: “participating in the developmental mathematics course has a positive impact on student retention [suggesting] to policy-makers that developmental education programs *can* be effective in helping to keep students enrolled in college” (Lesik, 2006).

Streaming

Although streaming was identified by only a few institutions (7/27), from subsequent site visits and conversations with mathematics educators of engineers the authors believe the practise to be common in Australian universities. Many questionnaire respondents did not view streaming as a “new” teaching and learning method. Streaming is an effective way to address the changing needs of engineering students, allowing mathematics educators to cover lower level mathematics with under-prepared students and to identify the gaps in their knowledge, while not repeating material that more advanced students have already grasped. Additional tutorials and increased office hours are often offered to students in the lower stream.

Students appear to be streamed into mathematics subjects in a variety of ways. The most common appears to be by the level of secondary school mathematics taken. However, with the increase in international students and the variations between state curricula, many institutions are finding it increasingly difficult to rely on students’ school results to allocate students to particular mathematics streams on entry to university so are introducing their own diagnostic tests for incoming students. These range from compulsory formal tests to informal voluntary online tests. The tests are often only used to advise, not to enforce entry to certain streams, allowing students the option to challenge themselves by entering a higher stream. Some institutions also spoke of encouraging students with weak mathematics backgrounds to take a lower level mathematics subject before attempting engineering mathematics subjects for their degree program, but spoke of the difficulty of getting students to heed this advice.

Two mathematics streams are available to engineering students at the University of Sydney. Students are encouraged to complete an online self-assessment [<http://www.maths.usyd.edu.au/us/selftest.cgi>] before deciding which mathematics stream to enrol in. This is however not enforced and students may select whichever stream they prefer. Britton *et al* (2007) made students enrolled in MATH1901 (the highest stream) sit the test in the first tutorial of semester. They found that the results in the test provided an accurate prediction of students’ performance in the subject. They found that the greatest number of incorrect answers came from questions that required conceptual knowledge and found that students often used algebraic procedure without reflection as to whether the answer is correct. Prerequisites for MATH1901 at the University of Sydney are HSC *Mathematics Extension 2* and a UAI of 90+. Results from the test seem to indicate that it is possible to achieve high marks in high level secondary school mathematics by performing mathematical procedures without understanding the concepts.

Curtin University of Technology streams students by the level of secondary school mathematics that they have taken, secondary school calculus no longer being a prerequisite for entry to the engineering degree program. Students with less than 65% in their secondary school leaving certificate enter *Engineering Mathematics* MATH120 and MATH140, while other students take *Engineering Mathematics* MATH110 and MATH130. Students in the higher stream have a wider choice of examination questions. All students must then take a common mathematics subject in second year.

In 1992 a regional university found that some engineering students were having significant difficulties with the first year engineering mathematics subject. In 1993 a new subject was introduced at the same level as secondary school intermediate mathematics for the minority who were unable to cope with the standard subject. By 1996 they found that very few students could cope with the original subject,

so abolished it. Currently in place is a minor modification of the second subject introduced in 1993 and a lower stream for students who are unable to cope, which includes Year 10 mathematics. The institution has found that many first year students are now commencing engineering degree programs unable to factorise integers and find lowest common multiples. These subjects have not been introduced as bridging subjects, in addition to the university level subjects; they now comprise the first of three engineering mathematics modules.

Bridging

The University of Adelaide offers three pre-tertiary bridging courses; *MathsBridge Online* - a web-based interactive course at year 10/11 level, useful for students to check their skills. *MathsStart*, a flexible delivery learning program, with work sheets to be completed at home and with workshops throughout the year, up to SACE Stage 1 level which is suitable preparation for *MathsTrack*. *MathsTrack* – in the same format as *MathsStart*, but to cover topics from SACE Stage 2. *MathsMate* is an evolving collection of mathematics online resources for students including interactive materials to refresh assumed knowledge; notes expanding on segments of the course which may be downloaded, as well as useful websites. For engineering mathematics subject *Mathematics 1/1M*, two modules may be downloaded covering the assumed knowledge for *Matrices* and *Systems of Linear Equations* [<http://www.adelaide.edu.au/clpd/maths/mathsmate/resources/download/maths.html>].

UTS offers free bridging courses for commencing students without the recommended knowledge for their degree program [<http://www.science.uts.edu.au/courses/bridging.html>]. *Maths (2 unit)* and *Maths Ext 1* run for two weeks, with daily 3-hour sessions either in morning, afternoon or evening. Students are provided with all course material. Similarly, the University of Sydney offer *The Extension 1 Course* or *The 2 Unit Course* [<http://www.maths.usyd.edu.au/u/BC/>] for students without the required prerequisites. Courses consist of twelve 2-hour sessions which are offered in either morning or evening.

UNSW operates the *Mathematics Bridging Course*, which is equivalent to Secondary school *Mathematics Extension 1*. Achievement of good grades in this bridging course allows students entry directly to MATH1131: *Mathematics 1A*. Students not deemed well prepared are advised to first complete MATH1011: *Fundamentals of Mathematics B*, which acts as a preparatory course, not a lower stream of MATH1131. While enrolment in this is highly recommended for under-prepared students, it is not enforced.

Alternative Entry

The University of South Australia offers a foundation program [<http://www.unisa.edu.au/study/progcourses/undgradpdf08/foundation.pdf>] which is designed as an alternative pathway for students to gain entry to a science or engineering degree program. The program runs full time for one year or part time for up to four years and is designed for people who have a gap in their science background. The program aims to determine suitability for entry to the program, but also to maximise the chance for success of those who are admitted. Analysis of a similar course run previously shows evidence that the foundation program has better equipped students for success in their degree program (Boland, 2002).

Swinburne University runs *Uniprep Science/Engineering*, a foundation program for science and engineering students [<http://courses.swinburne.edu.au/Courses/ViewCourseInternational.aspx?id=14089>]. The program runs for one academic year and covers a broad range of topics for students who must take *General Mathematics* and *Science Engineering Mathematics A & B*. The course prepares them for entry to first year.

Some institutions are creating outreach programs to attract students to science, engineering and mathematics. The University of South Australia operates Robotics Peer Mentoring (RPM) [<http://www.unisa.edu.au/mentor/robotics/default.asp>] with support from the University, industry and local councils. RPM provides hands-on experience in robotics, electronics, science and engineering for secondary school students. University undergraduate students act as mentors with secondary teachers to deliver a robotics program in schools, encouraging students to take enabling SET subjects such as mathematics by showing

them the relevance of such subjects later in robotics. As of 2007, over 1,000 secondary students will be involved in the program. Developing the links between the RPM program and industry and linking the program to key curriculum frameworks are goals to be achieved over the next few years. There has been a slight improvement in engineering enrolments in the last two years.

For a dual-sector University like RMIT, articulation pathways have been in place for some time for students transferring from TAFE to higher education. Specific exemptions have been negotiated for each of the main engineering schools. Students who have successfully completed diplomas and advanced diplomas will generally be awarded exemptions in credit points equivalent to a certain proportion of a year or years that make up the nominated engineering degree program. In many instances the selection officer for the engineering degree program would not include mathematics courses as part of the exemptions granted. However, the newly created Associate Degrees are different. These degree programs consist of higher education courses that are taught by TAFE staff and successful completion of such an award provide a guaranteed articulation pathway into the third year of a corresponding higher education engineering degree program.

Many engineering academics participating in the Carrick Institute of Learning and Teaching DBI *Addressing the Supply and Quality of Engineering Graduates for the New Century* commented that students find the transition from TAFE to an engineering degree program difficult due to the lack of mathematics in certificates and diplomas. Students interviewed who had gained entry through TAFE commented that mathematics is “daunting and difficult” (King, 2007).

4.3.5 Other Learning and Teaching Methods

Visual Worked Examples

Video resources have been introduced at the University of Wollongong to help address the insufficient knowledge of entering students to the first year mathematics program due to the decline in students’ basic mathematical skills. Sandison [<http://www.math.uow.edu.au/subjects/summer/refresh.html>] has developed worked video examples for *Summertime Maths* topic refreshers. These refresher subjects are available to all students online and comprise self-tests, theory refreshers, worked examples (both video and written), practice questions and links and text for further information. Topics include elementary algebra, algebraic fractions, Cartesian geometry, factorisation, logarithms, indices and elementary to advanced trigonometry. In these worked examples students watch the lecturer solving problems (with explanation) at the whiteboard.

Also at the University of Wollongong, Amnifar *et al* (2007) trialled two different methods for creating visual resources; video camera (a hand is videoed working through a mathematics problem and a recorded voice-over description is added) and eBeam (text appears “magically” on the screen as text is written on a connected touch pad and a recorded voice-over description is added). To ensure accessibility by all students from any location, video files were compressed and Mac and Windows compatible files were created. The files were made available to all students through webCT and listed according to mathematical content. The video camera method was believed to be the most successful as with this method the demonstrator was able to point to previous working to further explain the solution.

The resources were piloted in 2006. In student questionnaires at the end of the semester, 93% stated that the resources had helped them understand and learn mathematics. For those students who answered the questionnaire, results from two tests were analysed – a pre-semester 20-question multiple choice and a further 20-question basic skills test in week 4. For the pre-semester test, those students who indicated that they used the video resources initially averaged a fail grade (9.7/20) whereas those students who indicated that they did not use the video resources averaged a pass (12.6/20). Results from the second basic test in week four, after students had access to the video resources, showed that the weaker students averaged 11.95/20 and the stronger averaged 12.4/20 (no significant difference).

To assist students who enter engineering programs with insufficient mathematics preparation, the majority of whom have not completed a calculus subject at secondary school, the university of Western Sydney

has also developed video resources. These are to be used in conjunction with tutorial problems during semester. These online resources may be accessed by all students enrolled in the subject via WebCT.

Guest Lecturers

At some institutions, coordinators for mathematics subjects for engineering students invite guest lecturers to come in at various points throughout the subject to help students to see the relevance of the mathematics they are learning. The lecturers vary from professors from the engineering departments who present an example of the mathematics in an engineering context, highlighting to students the importance of mathematics for future studies, to industry professionals who explain how mathematics has been relevant and important to their career.

Continuous Assessment

Although only one institution identified continuous assessment as a new method of teaching and learning, of those institutions responding to question 5.2 *Which mathematics and statistics subjects for engineers use alternative assessment methods either in place or in addition to a written exam?*, all indicated that continuous assessment is used in engineering mathematics subjects. This is discussed further in section 4.4.

Re-sit Option

La Trobe University finds the resit option to be effective for students who fail tests during semester, providing them with additional opportunity to consolidate key skills. Teaching staff are available for one-on-one consultation with students to assist them in overcoming problems. The scheme appears to be working well, with positive feedback from students. There has been a noted improvement in the pass rate in Calculus A in first semester 2007, compared with 2006.

4.4 Methods of Assessment

With the introduction of flexible and varied learning and teaching at many institutions to appeal to the changing student body and appeal to different learning styles, some institutions are now investigating ways in which to make assessment less rigid and more inline with the learning and teaching experience students have in lectures.

Questionnaire responses indicate that some form of continuous assessment contributes to final mathematics grades for engineering students at all 27 of the responding institutions. Continuous assessment is believed to have various benefits and is viewed to be particularly valuable in subjects that require an understanding of foundation material for progression to later material in the subject. Continuous assessment is also believed to aid the transition for students from secondary school to tertiary education. Students often find this transition difficult, moving from small classes, with a large amount of individual attention to large lectures with often large tutorial sizes, with other students whom they do not know. They can find it intimidating to ask questions and may be unsure how to seek assistance with concepts that they are struggling with. Continuous assessment provides diagnostic feedback and allows lecturers and tutors to identify areas where either individuals or the entire group require further assistance, either individually or with the whole group in lectures or tutorials.

In question 4.2, we asked about assessment methods in the sense of summative assessment. Only 24 institutions completed question 4.2 *Which mathematics and statistics subjects for engineers use alternative assessment methods either in place of or in addition to a written exam?* The question also asked respondents to enter the percentage that each assessment method contributed to the overall final grade. The data collected is limited as the majority of respondents only answered for first year mathematics subjects. The histogram below shows the percentage of each student's final mark determined by the written exam. It may clearly be seen that continuous assessment is now commonplace in undergraduate mathematics for engineering students.

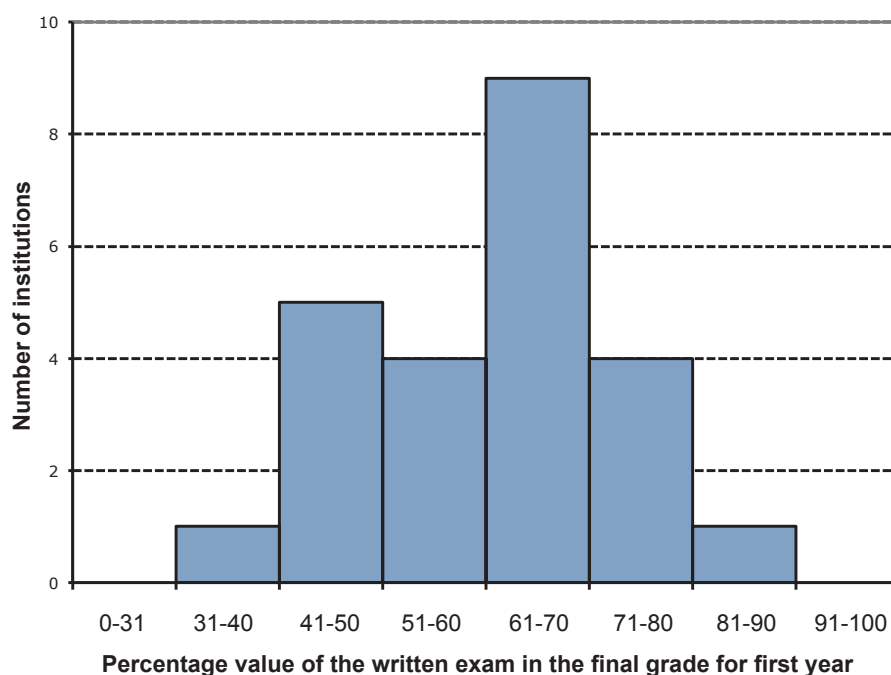


Figure 10: Questionnaire responses to question 5.2 Which mathematics and statistics subjects for engineers use alternative assessment methods either in place of or in addition to a written exam?

Respondents indicated that the additional percentage of marks is made up in a variety of ways at each institution, with little conformity between institutions. Additional assessment methods often include a combination of some of these assessment methods; class tests, group projects, individual projects, written assessment, mid-semester written exam and online testing or CAA.

While CAA is often used in summative assessment, there are clear benefits to the use of CAA in formative assessment. Many universities indicated that some form of CAA is available to students for this purpose, whether it be offered through commercial or in-house software. CAA is discussed in depth in Section 4.3.3.

Flexible assessment in tertiary mathematics is supported by Wood and Smith (1999), believing that giving students choice in their assessment forces students to take control of their learning and allows students to choose the method by which they are most able to demonstrate their achievement of course objectives. This increases equity between students, rather than favouring those who excel in examinations. When adopting this type of assessment approach it is important to ensure that assessment is a good reflection of a student's achievement of subject objectives. Wood and Smith (1999) cautions that too much choice at early stages of tertiary education can add to the level of complication. Students need support in the transition from secondary school to university, but a limited amount of choice can be very successful. The example was given of a first year lecturer who allowed students to choose 1, 2 or 3 assessments worth 20% and an examination with 4 questions worth 20% each – students had to decide and submit their choice by week 4.

Cretchley (1999) is also a proponent of the use of flexible assessment with student self-choice in early undergraduate mathematics assessment, believing it to be an effective way to deal with some of the challenges faced by mathematics educators. Student self-choice gives students the opportunity to select assessment tasks that suit their learning style, to stretch themselves within their comfort zones and to focus on their interests and needs, particularly in first year undergraduate mathematics where various disciplines are all taught in communal mathematics lectures, with students with very differing mathematical needs and backgrounds. Through self-choice students are able to focus on the topics which are of particular relevance to them. Some compulsory questions were also set as guidance for students. While student feedback was largely positive in the first trial of this method of assessment, staff struggled with the burden of the extra marking.

To aid the mathematical confidence of first year engineering students at QUT a system that combines flexible, formative and summative assessment was introduced in 1999 (Cuthbert and MacGillvray, 2003). Assessment is divided into three sections. Students may choose to sit tests in weeks 5, 9 and 13 of semester, or to be tested on similar material at the end of semester. Students may choose to take the semester assessment marks or sit the end of semester tests, with their best results in each section used. Weekly tutorials also have a 5-10 minute test, results of which may be used towards the final result. Most students attempt the test during semester and weekly tutorials have very high attendance. This assessment process has received very positive feedback from both students and staff.

4.5 Subject Content and Organisation

The Engineers Australia accreditation system for degree programs is fully compliant with the Washington Accord (see <http://www.washingtonaccord.org/Washington-Accord/Accredited.cfm>). However, the current version of the Engineers Australia Accreditation Criteria Guidelines (Engineers Australia, 2006b) gives no specific advice on mathematics content. In those guidelines, Section 3.2.3 Program Structure and Implementation Framework says that Australian 4-year professional engineering degree programs should include the following “indicative components of the total learning experience”:

- “mathematics, science, engineering principles, skills and tools appropriate to the discipline of study (not less than 40%),
- engineering design and projects (20%),
- an engineering discipline specialisation (approximately 20%),
- integrated exposure to professional engineering practice, including management and professional ethics (approximately 10%),
- more of any of the above elements, or other elective studies (approximately 10%).”

The only other reference to mathematics is made in Section 3.2.4.1 Enabling Skills and Knowledge Development:

“Enabling skills and knowledge in mathematics; physical, life and information sciences, and in engineering fundamentals must adequately underpin the development of high level technical capabilities, and in engineering application work within the designated field of practice and selected specialisation(s).”

While the great majority of engineers that we contacted felt there was a need for some mathematics education, there is no general agreement on any minimum mathematics component within the specified 40% on mathematics, science and other enabling subjects. Despite this degree of flexibility, there was widespread agreement among the universities on core mathematics content at first year level, partly because of Australia’s long-established traditions in engineering science. During 2007, parallel to our own investigation, S. Barry and L. Healy of the Australian Defence Force Academy (a Canberra-based College of the University of New South Wales), compared the mathematics content in engineering programs at nine Australian universities, including representatives from six of the eight main Australian states and territories (Barry and Healy, 2007). Having communicated with a large number of universities, we can agree in broad terms with their finding of widespread uniformity in the coverage of the topics of

- Calculus (introductory and one-dimensional)
- Linear algebra
- Multivariable calculus
- Ordinary differential equations
- Introductory statistics

In Australia, introductory calculus is not taught as a separate subject but is commonly integrated with some topics in algebra, probability, discrete mathematics, numerical analysis, and complex analysis. The amount of material in these last four topics varies between institutions. In order to accommodate a more diverse student group and a reduction in the number of teaching weeks (down to 24 weeks per year at some institutions that 15 years ago had 28-30 weeks), several institutions had removed some

material, most commonly in the areas of discrete mathematics, numerical analysis and complex analysis. Since there are many opinions on the relative priority of these topics, some non-uniformity between institutions is developing. Barry and Healy (2007) found some uniformity of the content in introductory probability. However, several engineering departments and several current engineering students have stressed the need for more probability and statistics, in response to modern engineering practice, in areas such as maintenance planning, quality monitoring and stochastic modelling of noisy communication networks (King, 2007, MacGillivray, 2007). At on-site visits, several staff lamented on the weakening of complex analysis, to the point that it had become more difficult to teach frequency-amplitude-phase analysis in engineering courses involving electrical and mechanical oscillation phenomena. At one university, engineers and mathematicians jointly teach a successful first-year elective course in mathematical modelling and numerical analysis. This course is popular among engineering students partly because it is jointly taught by engineering staff who consequently recommend it.

Barry and Healy (2007) found some uniformity in the mathematics material covered generally over the first two years of degree programs. However, their sample of institutions included mostly the larger capital city universities. Only two universities in their sample of nine did not run a compulsory mathematics subject in Semester 4 of the program. We have found that the smaller universities tend to run fewer compulsory mathematics subjects and consequently their coverage is not as broad. We heard from several mathematics departments that by the time they finished teaching their engineering students, they had not been introduced to partial differential equations. Many agree that it is important for example, for mechanical engineers to know that the system of Navier-Stokes equations is the accepted model for many fluids and that these are the main objects of interest for computational fluid mechanics software packages often used in professional practice. At many institutions, we are relying on engineering staff to teach these important mathematical concepts. Some of them, particularly those who had been well trained in the past by specialist mathematicians, are well qualified for this task.

For each of the universities surveyed, we first consulted their publicly available subject outlines and made a preliminary identification of those subjects that appeared *prima facie* to spend at least 50% of class time on topics in mathematics and statistics, and which were commonly taken by engineering students. The listed subjects included those traditional subjects in engineering that have high mathematical content (e.g. fluid mechanics, finite element methods, control theory, signal processing). The questionnaire asked the respondents to verify which of the listed subjects contained at least 50% mathematics content and to list others that we had missed. They were also asked to tick boxes to indicate which specialisations of engineering took those courses and to give typical class sizes for lectures, tutorials and laboratory classes in those subjects.

At first year level, class time typically consisted of four hours of lectures and a one-hour tutorial per week. There was some variation in the number of lectures, three hours per week in several programs partly compensated by computer laboratory sessions or group-work project sessions. The distribution of average first year and second year lecture sizes across the universities is given in the following figure:

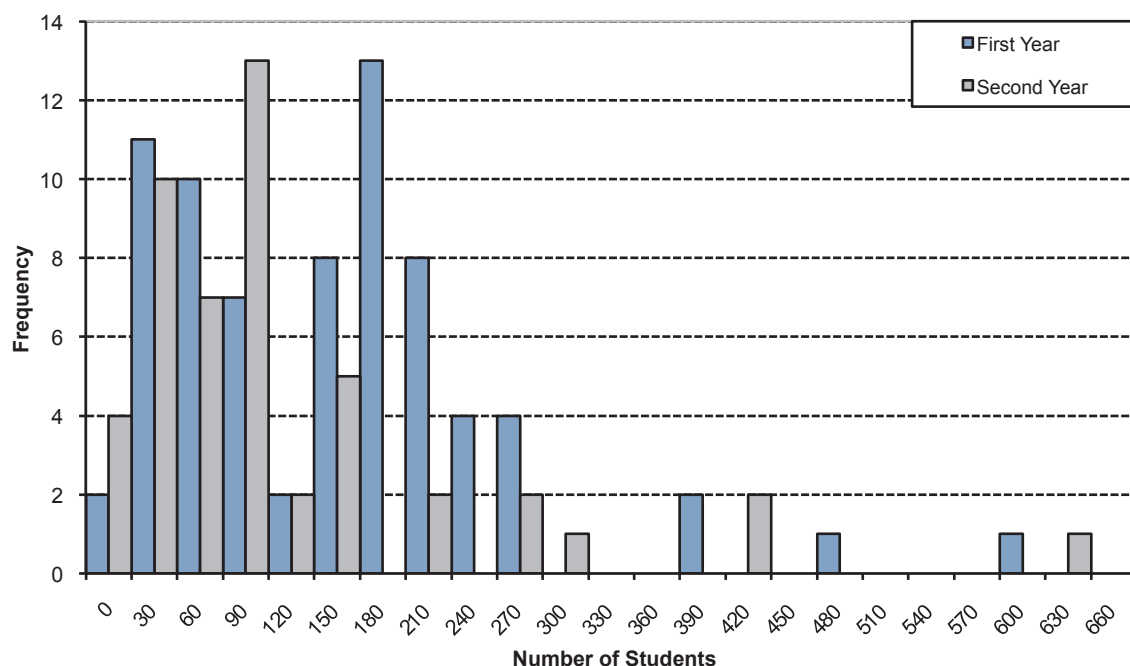


Figure 11: Questionnaire responses to question 1.4 List the typical number of students that attend a single lecture for each subject listed.

Ten universities conduct mathematics laboratory classes for first year engineering students. Class sizes are determined almost exclusively by the number of computers or terminals in a room. Two had laboratory class sizes of around 50. Two conducted laboratories only for project groups of 5-6. All of the others had class sizes in the range 16-25.

The distribution of tutorial class sizes in first and second year is given in the following figure:

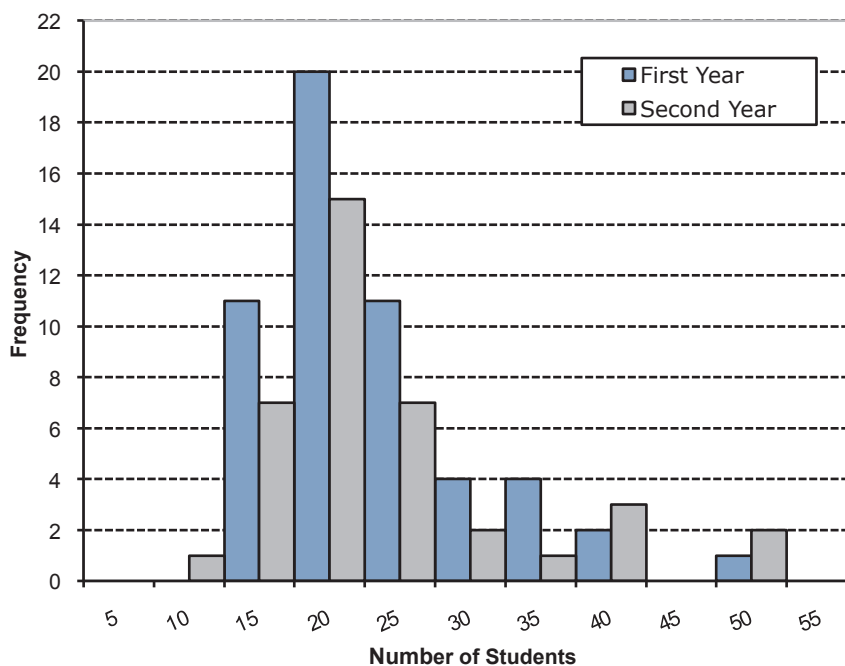


Figure 12: Questionnaire responses to question 1.4 List the typical number of students that attend tutorial class for each subject listed

Only one university does not run a compulsory tutorial. Their students receive plenty of practice on regular randomised assignments that are submitted using and assessed by a well designed computer system.

Although in our site visits, we found widespread recognition that different specialisations of engineering favoured different mathematics topics, distinct first year mathematics courses were offered to distinct specialisations of the BE degree in only three institutions. King (2007) found that civil engineering academics in particular felt that their students did not need the same level of mathematics as other specialisations of engineering. At some institutions, mathematics departments offered the same first year mathematics not only to engineering majors but to all mathematics majors and other science majors as well. One significant motivation was a changing government competitive funding system that encouraged universities to produce more research outputs. Many universities allocate no undergraduate teaching to their strongest researchers, thereby adding extra load to the teaching staff. One mathematics respondent admitted that there was a plan to rationalise first year mathematics offerings because of the pressure to perform better at research.

Nine universities had at least one specialisation of a bachelor of engineering degree for which students had to take two or more second year subjects with more than 90% mathematics content. The average number of second year mathematics subjects was 1.5. This figure does not include the two second-year mathematics subjects that are “highly recommended” but not compulsory for engineering students at one of Australia’s larger universities. Although that is the only university that has zero mathematics requirements at second year level, the majority of its students take both second year mathematics subjects and their engineering subjects tend to have high mathematical content. The average of 1.5 subjects does not include the additional second-year applied mathematics subject that is compulsory at one institution for engineering-mathematics double degree students. In some of the larger sandstone institutions where entry to engineering requires high tertiary entrance scores, up to 50% of engineering students enrol in double degree programs with Economics, Business Management, Computer Science or Mathematics.

One university had accommodated the extra demand for statistics by adding a statistics half-subject to its two pre-existing compulsory second year mathematics subjects. Another had accommodated that demand by removing any material on partial differential equations from its sole compulsory second year subject.

Question 3.4 asked, “*Do all engineering students gain the same foundation knowledge of mathematics/statistics? Please explain.*”

Some universities identify a subgroup of incoming at-risk students that is then required to take an additional bridging course or foundation course either in the summer break or during normal semesters. At one university, this subgroup is identified simply as those who “for one reason or another” are admitted without having satisfied the formal mathematics entry requirements. Three engineering programs offer two different streams of mathematics at first-year level, depending on the preparedness and/or motivation of the students. However, in all cases the contents of courses for both streams are meant to be adequate preparation for second-year courses taken by both groups. Several respondents said that all forms of additional learning assistance have the explicit aim of preparing all students for common courses at second-year level.

Eight of 24 respondents could identify specialisation-specific mathematics subjects at second year level. Of these, eleven offered specialisation-specific subjects devoted to mathematics, while the others were specialisation-specific engineering subjects with high mathematical content. In one institution, there are parallel mathematics subjects for three different specialisations, even varying markedly in the mode of delivery. One of these adopts a problem-based learning mode and the other two are more traditional. In some cases, the number of compulsory mathematics subjects varies between specialisations, electrical engineering on average taking more mathematics.

One university has an additional combined mathematics-physics course at second year for electrical engineering students. It is compulsory and jointly taught by staff from the physics and mathematics disciplines. Satisfaction with the subject is reported by both students and staff.

At third and fourth year levels, the information that we received was less complete. We learnt from site visits that engineers were reluctant to divulge the mathematics content of their courses because

of perceived take-over threats from other disciplines. This issue will be discussed further in Section 6 Oversight and Organisation.

Eight universities offered subjects taught by mathematics staff specifically for engineering students at third-year level, and two other universities offered such subjects at fourth-year level, without having pre-requisites in third-year mathematics. Some of these more advanced subjects were specialisation-specific, for example coding and cryptography for computer engineering, and mathematics for communication engineering. In other cases, these advanced courses were general in content, and optional.

4.6 Oversight

Where engineering students take compulsory mathematics courses taught by a mathematics department, the latter is often viewed as a service provider and the engineering faculty is viewed as a client. Unlike in most provisions of service, in this case the service provider and client have very similar status, roles and job functions, even sharing the same employer. Therefore, there is the possibility of various degrees of sharing of responsibility for the curriculum design, pedagogical strategy, review of student learning outcomes and course quality review.

In our experience, there is a wide diversity of opinions among academic staff on what should be included in the curriculum and on what degree of mathematical rigour students should be expected to develop. Each branch of engineering has its own preferences for mathematical topics. However, mathematics departments rarely have the resources to offer separate mathematics courses for each branch of engineering. Therefore some decisions must be made on compromises in curriculum design. Client satisfaction is more likely when the clients have participated in the decision-making process, so that they feel some level of joint ownership of the product.

Responses to related questions in the questionnaire were varied, and difficult to categorise in tabular form. However, we will quote some of the interesting responses, and make some general comments that were partly informed by site visits.

Question 4.3 of the questionnaire asked, *“Which parties (engineering staff, mathematics staff, students, Department of Teaching and Learning, industry partners, etc) are involved in the design of mathematics and statistics subjects for engineering students?”*.

Most responded that the curriculum is decided by a group of mathematics staff in consultation with the engineering staff. Some admitted that this consultation is informal. In some cases, there is a formal committee to set the curriculum. On each of these committees there are at least two mathematics staff (except at the university that doesn’t have two mathematics staff), and at least two engineering staff. One representative from Mathematics admitted,

“...we have liased (sic) with Engineering staff on a regular basis but the communication channels are dreadful. The views we get change radically from year to year as the person (junior staff) in Engineering who liase with us changes.”

In two universities, advanced mathematics courses were offered by mathematically trained engineering staff, for example in advanced systems modelling. We met several engineering staff who were qualified in mathematics. We did not meet engineering staff who were teaching mathematics subjects yet had not themselves been taught to an advanced level by mathematicians. Many engineers were reluctant to divulge which of their engineering courses contained 50% or more mathematical content. At site visits, we learnt that engineers perceived some threat of losing control of some subjects to another department. At some institutions, there is some tension between the teaching departments over who should teach what. Predominantly service-teaching disciplines such as mathematics are sensitive about control over their core business. It seems that the government’s centralised differential funding system does not encourage interdisciplinary team teaching. Quoting one correspondent mathematician:

“Our engineers are coached on how to write their basically maths units as engineering units so as to get the better funding.”

It was made clear to us that considerable mathematical material is learnt within traditional engineering courses. Some of the engineering educators explained that their emphasis was more on practical techniques than on mathematical rigour. This was expected from the outset and it was a practice that should be continued; otherwise mathematics would not be used effectively.

At one major engineering university where the Mathematics department is part of the Engineering Faculty, the mathematics respondent wrote,

“Generally, our engineering colleagues trust us to deliver high quality teaching that is relevant to their areas. We do not generally get complaints. ”

Another major engineering university had a formal engineering mathematics committee that had a representative from each engineering department, outnumbering the two representatives from Mathematics.

At one university, a series of subject design meetings between engineers and mathematicians was presided over by an independent Dean before both disciplines fully agreed. Since then student and staff satisfaction with the mathematics subjects has increased dramatically.

Question 4.2 asked, *“Who/which department(s)/committee/centre is responsible for overseeing the teaching and learning of mathematics and statistics subjects for engineering students?”*.

Almost all of the respondents identified a responsible group but those groups had various compositions.

One major university holds regular feedback meetings involving all mathematics and engineering departments. Feedback is provided during the session by the central education development unit that convenes focus groups of students. At one of the technical universities, in the last week of session the central education development unit holds a pizza lunch at which first-year engineering students are invited to comment. Information is then conveyed to the unit panel, thence to the Engineering Faculty's first-year review panel, for discussion. Another technical university identifies a mathematics service team that has

“developed and maintained strong ties with their compatriots in the Engineering schools to the extent that we are now deemed to be part of their own Program teams”.

At that institution, program leaders meet monthly at portfolio meetings where educational progress is discussed.

During one site visit, a mathematics department education committee was reviewing its undergraduate offerings, with each committee member asked to gather feedback from each client department, including each engineering department.

One engineering representative wrote,

“Engineering Program Convenor attempts to interact with service provider”.

Question 5.3 asked, *“How is subject content evaluated? (e.g. staff feedback, student feedback, graduate exit survey, industry feedback). Please explain”*

There were few surprises in the responses. All universities have a mechanism for review of class results. All have a regular instrument for student satisfaction surveys applied to all subjects but individual teacher effectiveness surveys are not public. In several cases, student performance and satisfaction data at the subject level, are reviewed by the coordination committees referred to in Question 4.2. In some cases, there was no regular systematic meeting for such reviews. In those cases, Associate Deans, Department Heads and Year Coordinators were responsible for feeding information back to lecturers, and for suggesting means of quality control. In one institution, there were regular “Cross-year” reports in which lecturers comment on preparedness of their students who arrive with earlier subjects completed as prerequisites. In other institutions, discussions on these matters occur incidentally in curriculum committees.

Some departments found the 5-year department reviews and Engineers Australia accreditation reviews as being particularly helpful in their provision of external observations, comparisons and advice.

Three of the technological universities reported that industry representatives and recent graduates on external advisory committees were able to comment on teaching and learning outcomes.

One engineering department surveys all of its graduates and their employers every year.

In response to the question 6.1 *How has your mathematics and statistics curriculum for engineering degree programs changed in the past 5 years in response to changing requirements of employers/ perceptions of employment opportunities?* most institutions states that there had been no change to the mathematics curriculum for engineering students in response to industry requirements.

One institution referred to the lack of communication between the engineering department and the mathematics department

“It hasn’t, we have had no industry feedback. The Engineering departments have strong industry connections, but we are not in the loop and suggestions are not passed on to us.”

Three institutions referred to an increased use of computer software that industry requires students to be familiar with.

Two institutions referred to introducing PBL and teaching in context in response to perceived industry and community requirements for increased levels of “soft” skills in engineering graduates. One institution spoke of moving to more specific mathematics requirements for each discipline.

Only one institution spoke of an increase in mathematics content in response to industry requirements:

“Probability and Stats has been increased to 50% of a second year unit.”

Trevelyan (2007) found that engineers in the workplace are reluctant to use mathematical and analytical techniques learnt at university. This needs to be addressed, and engineers need to graduate comfortable with using mathematics in every day tasks. Referring to the economic benefits to employers of employees who are able to make more accurate predictions of performance and the financial benefits to employers of the reduction of uncertainty, the author also states that “engineering science almost entirely depends on mathematical analysis and representation and engineering science lies at the heart of both accurate prediction and the reliable delivery of practical solutions” (Trevelyan 2007).

The First Year

Backed by continuing research there is increasing recognition of the importance of the first year for undergraduate students and the effect of this year on attrition rates and the over all university experience. Krause *et al* (2005) found that the majority of school leavers do not believe that their final year at secondary school was good preparation for their university study and that university study did not build on what they have learnt at secondary school.

With the increasing demand for engineers, changes to the higher education funding system and increased accountability of education providers, there are strong incentives to address attraction and retention of students. At the same time, the system has to cope with a decline in the number of Year 12 students taking higher level mathematics, a flattening of the number of domestic students attracted to engineering, an increasingly diverse (both academically and demographically) student body and a large amount of foundation material to be covered in first year. This all contributes to the first year being of particular importance in undergraduate engineering degree programs.

McKenzie and Schweitzer (2001) examined the major predictors of academic success in tertiary education identified in previous research and found that study skills, integration into university, financial situation, career orientation and social support are all predictors of university performance in first year. This further highlights the importance of implementing a holistic approach to the transition from secondary to tertiary education.

“The first year program [for the undergraduate engineering degree program] serves a number of purposes which typically include: a cultural and social transition for school leavers into higher education; the laying of the academic foundation upon which the individual grows and attains the desired

graduate attributes as set by the program and by institution and professional accrediting bodies; and the beginning of formative development as a professional and citizen of tomorrow.” (Campbell *et al*, 2007)

A number of universities are attempting to address these issues, for example *The First Year Experience Project* at UQ, First Year Engineering Review (FYER) at QUT, the development of the Engineering Foundation Year (EFY) at CUT, the adoption of the Melbourne Model in 2008 at the University of Melbourne, and the Plan for First Year Academic Orientation and Transition at Griffith University.

Internal research at QUT has shown that the first year of an undergraduate degree program can have the most significant impact in terms of student experience, with evidence indicating that many students’ enthusiasm for the program is dampened by having to sit through more traditional classes in basic sciences (Campbell *et al*, 2007). In addition to this some mathematics lecturers noted that most failures in first year mathematics were attributed to students who miss the majority of scheduled classes.

Many of these reports have highlighted the importance of interdepartmental cooperation and collaboration to provide a coordinated approach to identifying struggling students, provide additional support for students requiring it and improve the overall first year experience for students. Many respondents to the questionnaire also indicated that much of the extra mathematics support offered to students was targeted at first year students.

The *Engineering Foundation Year* was developed at CUT and it provides a good example of an effective method to address some of the challenges faced by universities. The EFY was developed to ensure that students gained the foundations for later discipline specific study in their engineering degree program, through a learning experience reflecting engineering practice and to aid the transition from secondary school to university. The EFY aims to give students experience in engineering fundamentals – competence in mathematics and computing, understanding the relationship between science and engineering, learning skills and professional practice.

The curriculum comprises a set of inter-related units:

Engineering Foundations – design skills, creative thinking processes, the principles of engineering and the ability to communicate,

Engineering Science – the traditional subjects, the laws of physics and chemistry

Enabling Skills – mathematics and programming which provide the means by which engineers solve problems and provides the structured thinking.

The mathematics curriculum for the EFY was designed through a series of meetings held by the EFY committee and engineering mathematics subject co-ordinators and chaired by the Dean of Learning and Teaching to develop a common understanding of the aims of engineering mathematics, giving mathematics and engineering staff joint ownership and agreement on the curriculum, with both parties having a clear understanding of what students will learn. Students are streamed into two groups according to their secondary school results, with both strands achieving common competencies. Engineering mathematics now gets a strong evaluation from students. In 2007 it was higher than for engineering subjects – 93% positive.

The EFY Studio provides learning facilities that engage students with the engineering profession, reflecting the layout of a modern office. It provides students with a communal area to study and project rooms for group study. Mathematics support is also offered in the EFY with lecturers and tutors available for a specified number of hours each week for any student requiring assistance. The Dean of Learning and Teaching holds student feedback sessions at the end of each semester to identify and deal with any issues that may arise.

Many other institutions are implementing programs and support networks to aid the transition from secondary school to university, CUT provides an example of a holistic approach which has been seen to be effective with positive feedback from both students and academics.

4.7 International Site Visits

4.7.1 United Kingdom

In the UK, the traditional pathway to a professional engineering career is a three-year first degree followed by a two-year masters degree. British academics have been involved in the European Society for Engineering Education (SEFI) that gives a clear guide to the mathematics curriculum (e.g. SEFI Working Group, 2002).

For example, in the core material to be learnt by all engineers, the following material is specified on the topic of complex numbers:

“As a result of learning this material you should be able to

- state and use Euler’s formula
- state and use de Moivre’s theorem for a rational index
- find the roots of a complex number
- link trigonometric and hyperbolic functions
- describe regions in the plane by restricting the modulus and/or the argument of a complex number.”

Since 2006, accreditation of engineering degrees in UK has been controlled by the Engineering Education Board (EAB). EAB specifications of mathematics learning outcomes are not nearly as detailed as those of SEFI but they are a little more detailed than those of Engineers Australia. For example, in the sample specification for the BEng(Hons) in Chemical Engineering:

“Knowledge and understanding of mathematical principles necessary to underpin their education in their engineering discipline and to enable them to apply mathematical methods, tools and notations proficiently in the analysis and solution of engineering problems.”

Loughborough University

The University originated from a small technical college, becoming Britain’s first technological university in 1966. It now has around 12,000 students. The Engineering Faculty has 3000 students and the estimated intake is 800 students each year (Times Online, 15 August 2007 http://www.timesonline.co.uk/tol/life_and_style/education/good_university_guide/article2132360.ece).

Loughborough University is widely recognised as a world leader in mathematical education for engineering students. The University recognizes that mathematics education of engineers is an important and exacting task and that it is worthwhile to hire good people for that task and to provide the best available resources. This investment has been rewarded by additional government funding. The University is the lead organisation for 2 of the 74 Centres of Excellence for Teaching and Learning recently set up by the Higher Education Authority. These are the Engineering Centre for Excellence in Teaching and Learning [<http://www.hefce.ac.uk/learning/tinits/cetl/final/show.asp?id=23>] and SIGMA - Centre for Excellence in Mathematics and Statistics Support [<http://www.hefce.ac.uk/learning/tinits/cetl/final/show.asp?id=24>].

The Sunday Times Online has ranked Loughborough University as 13th among universities in the UK. Rankings for specialist engineering programs at Loughborough University include: civil engineering at 11th, electrical engineering at 12th, and mechanical engineering at 12th [www.timesonline.co.uk].

A complete mathematics subject content for two engineering disciplines, electrical engineering and mechanical engineering, is provided in Appendix 1.

Mathematics Education Centre (MEC) at Loughborough University

The Mathematics Education Centre (MEC) manages teaching of mathematics subjects to engineering undergraduate students at Loughborough University. The centre supports Mathematical Learning Support Centre (MLSC), which offers support to any students needing help with basic mathematics. Students benefit from free resources, tuition and short courses in a drop-in basis. There are two drop-in

centres, one housed in the MEC building and the other in the Mathematics building. Additionally MEC provides extensive mathematical support through HELM (Helping Engineers Learn Mathematics) network.

HELM

The HELM project, undertaken by five UK universities and led by Loughborough University, aims to enhance mathematical education of engineering undergraduate students, by providing a range of teaching and learning resources. These resources are available not only to the five institutions in the network but to any other institution that pays an annual registration fee.

HELM resources include workbooks, web-delivered courseware and associated computer assisted assessment. The workbooks contain mathematical topics and exercises written specifically for engineering undergraduate students, and contain mathematics and statistics materials. These workbooks are world renowned because they are comprehensive, covering most conceivable topics at all levels of undergraduate engineering mathematics. There are also books containing a selection of Mathematics and Physics related problems, and advanced engineering case studies. In addition there is a student's guidebook and tutor's guidebook [<http://mlsc.lboro.ac.uk/helm.php>].

LTSN MathsTEAM

Funded up to 2007 by the Learning and Teaching Support Network (LTSN), the LTSN MathsTEAM is a collaborative project between four subject centres (LTSN Maths, Stats & OR Network, LTSN Engineering, LTSN Physical Sciences and the UK Centre for Materials Education).

The LTSN MathsTEAM recently surveyed the growing number of innovative teaching methods throughout the UK. Three booklets (listed below) have been published, each providing a comprehensive collection of case studies which describe the execution of the learning activities, the support needed, the implementation, the difficulties and evidence of success.

- Maths for Engineering and Science (LTSN MathsTEAM, 2003a)
- Diagnostic Testing for Mathematics (LTSN MathsTEAM, 2003b)
- Maths Support for Students (LTSN MathsTEAM, 2003c)

4.7.2 USA

In the USA, most BE programs are of four years duration, including a number of breadth requirements for such areas as writing, foreign languages and cultural awareness. Most programs contain a significant number of mathematics requirements partly because the main accrediting agency ABET provides program criteria with explicit mathematical content. The criteria for electrical engineering programs include,

"Programs containing the modifier 'electrical' in the title must also demonstrate that graduates have a knowledge of advanced mathematics, typically including differential equations, linear algebra, complex variables, and discrete mathematics."

The 2007-08 curriculum criteria for mechanical engineering include,

"knowledge of chemistry and calculus-based physics with depth in at least one; the ability to apply advanced mathematics through multivariate calculus and differential equations; familiarity with statistics and linear algebra".

Most programs include five or six compulsory single-semester mathematics subjects. However the first three of these are often introductory calculus, a second course in calculus, and multivariate calculus. Most engineering students have at least one of these courses waived after taking one or more of the subjects Advanced Placement Calculus 1,2,3 offered nationally to high school students. The current popularity of these courses forces many universities to adopt similar syllabi so that they can attract better students with credit for AP subjects. The fourth subject is traditionally a combination of differential equations and linear algebra and the fifth usually involves partial differential equations and/or numerical methods and/or statistics.

Duke University

This is a private university, formed in 1926 but having evolved from a college that originated in 1838. It has only around 6000 highly selected undergraduates but there is a higher number of students enrolled for postgraduate professional and higher degrees. Its most highly acclaimed engineering department is Biomedical Engineering, for example, ranked fourth nationally by Infozee [<http://www.infozee.com/channels/ms/usa/branch-rankings.htm>]

Duke University's Pratt School of Engineering offers the Bachelor of Science degree in four major engineering disciplines. All majors lead to the degree of Bachelor of Science in Engineering (BSE). Duke University's engineering program is unashamedly science - based.

Engineering students complete 4 basic mathematics subjects including *Calculus 1*, *Calculus 2* and *Intermediate Calculus* which are taught in laboratory calculus courses, as well as *Linear Algebra & Differential Equations*. At the request of the Engineering faculty, the course in Linear Algebra and Differential Equations has a primary emphasis on Linear Algebra, with systems of ODEs as an application, rather than the other way around which is the conventional course in most American EE degree program.

The separate course on Probability and Statistics replaces some material on numerical analysis, which is more common in American programs. Again this decision was made after close consultation with the Engineering Faculty. The course has proven to be popular among students.

A complete mathematics subject content for two engineering disciplines, electrical engineering and mechanical engineering, is provided in Appendix 1.

The Department of Mathematics prides itself on being responsive to the expressed needs of the Pratt School of Engineering.

Laboratory Calculus Courses

Laboratory calculus courses were designed by the Department of Mathematics at Duke University in 1997 and have been active since then. They do not use specialised rooms. The laboratory work is done with scientific graphing calculators. Compared to traditional calculus courses, they focus on concepts and applications, rather than on mathematical rigour or on developing integration techniques. Students are graded on major tests, as well as lab report, lab quizzes, and homework quizzes (Blake and Reed, 2004).

Unlike in many other universities, they use calculators as their only technical aid. This means that the students are not distracted by learning a new programming language. In American high schools, students are used to using graphics calculators on a day to day basis.

The weekly calculus classes include three of 50-minutes and one laboratory class of an hour and forty-five minutes. During the class time the instructor will supervise group work and discussion, as well as give some explanations in a lecture format.

University of Delaware

The University of Delaware evolved from a college that originated in 1743 [<http://www.udel.edu/>]. It is a state-supported university but acts in some ways as a private institution; the State does not appoint members to its Board. The College of Engineering has around 1300 undergraduates and 500 students in professional and higher degree programs. The highest ranked engineering department is Chemical Engineering, for example ranked 9 nationally by Infozee [<http://www.infozee.com/channels/ms/usa/branch-rankings.htm>].

A complete mathematics subject content for two engineering disciplines, Electrical Engineering and Mechanical Engineering, is provided in Appendix 1.

The MEC Lab (Modeling, Experiment, and Computation)

The MEC lab is an experimental mathematics laboratory, where undergraduate students are involved in hands-on learning experience and mathematical modelling. The lab is not a computer lab; it is a wet or physical lab [<http://www.math.udel.edu/MECLAB/>].

Regular and innovative courses, as well as research projects for both undergraduate and postgraduate students, run in this lab. Math 512 *Math Modeling* is one of the innovative courses, which is based on teamwork and problem-based techniques. This course is offered to mathematics majors but is very popular among the engineering students. It is a project-based course where students take up one project and work on it throughout the entire semester. Report writing is a significant component, so that completion of the course satisfies the University's writing requirement.

The content of course covers mathematical topics including ordinary and partial differential equations, systems of differential equations, transforms, asymptotic and numerical methods. Prerequisites are a 300 or higher level course on differential equations.

The course is designed for group work and focused on building on teamwork and, improving students' speaking and writing skills. In the lab, students are able to assemble objects and watch them interact, take video recordings and formulate mathematical models. Students work on problems and some of the problems are associated with local companies. The final product of this class is a journal style paper, which is continually assessed and graded throughout the semester.

There is another approach to lab courses, where students still work in teams, as in the problem based learning technique. But the difference is that students work on a sequence of classic problems. This approach focuses on recreating classic experiments and reproducing classic mathematics, which could result in re-creation or something novel.

Interdisciplinary undergraduate research

In some project-based learning activities, students from engineering or mathematics work in research teams with students from other disciplines. For example, in the MEC lab, mathematics students in MATH512 work on the same projects with other students in a food technology subject.

4.8 Some Overseas Comparisons

The USA engineering accreditation board ABET and the European accreditation board SEFI are more definitive than the Australian accrediting body EA in specifying mathematics content in their criteria for program accreditation. Australia has not investigated how much influence SEFI has in the shaping of European and British curricula. However, ABET has considerable influence in USA engineering programs. Compared to their Australian counterparts, USA engineering students expect to spend more hours on subjects that concentrate on mathematics. Accreditation criteria of the British EAB are not so specific on mathematical content. Australia has not investigated the degree of variability of mathematics content in British engineering programs.

In the UK, there seems to be more general awareness of the problematic issues surrounding the attraction and retention of students in science, education and technology and on ways of improving their scholastic performance. Australia has no government-funded project comparable to HELM that was funded over a two year period by the British government with the sole aim of helping engineers learn mathematics. Australia has no organisation that disseminates such a wide variety of high quality teaching materials. In the recent past, there have been several British government-funded programs to support educational developments in these areas. The most prominent of these is the group of Centres for Excellence in Teaching and Learning, administered by the Higher Education Authority. So far, 74 Centres have been set up. Each of these has a lead institution but in many cases they involve networks of several institutions. The DBI initiative of the Carrick Institute has been a very significant development but not at the same magnitude.

In the USA, the independent Carnegie Foundation for the Advancement of Teaching has been instrumental in encouraging new ideas. Its concept of a capstone course has been most influential (Boyer Commission, 2007)

In this model, the final year of a degree program includes a capstone course that draws on the students' experience from a number of earlier subjects. It often involves some interdisciplinary investigation. In the Delaware MEC Lab, it has been found beneficial to combine an applied mathematics capstone course

with that from another discipline. In Australia, there are many interesting multidisciplinary projects involving engineering students but the project has not discovered any involving mathematics majors. One of the limiting factors is the small number of mathematics majors. Across the OECD, 1% of university graduates have a mathematics major but in Australia, the figure is 0.4%.

Some Australian universities have a long tradition of excellence in engineering education. Some have set up innovative teaching and learning areas, sometimes assisted by sponsors in the engineering profession. However, in the current funding climate it is difficult to imagine an Australian university devoting as many resources to undergraduate mathematics and statistics support as has Loughborough University. Many American universities have large privately funded endowments from which they may support special initiatives in education. The UNIDEL Foundation helped to set up the MEC Lab in the Mathematical Sciences Department at University of Delaware. In USA, several mathematics departments have physical/chemical labs for undergraduate project work but there are none planned in Australia.

The National Science Foundation of the USA is able to fund Research Experience for Undergraduates as an item of budget within large research grant applications. In Australia, the Australian Research Council cannot fund undergraduate educational activities even when they relate to a research project. In fact the nexus between teaching and research is weakening because the current trend in universities is to remove the strongest researchers from the classroom so that they can devote themselves to the business of earning research income.

5. The Current State of Engineering Mathematics

Professional practice subjects such as project management, business management and finance are included as compulsory subjects largely at the expense of basic science subjects. While much mathematics is used indirectly in professional engineering practice there is growing concern that the majority of professional engineers in Australia are not confident in their mathematical abilities and consequently use little mathematics in their careers. While this is the case the majority of academic and professional engineers surveyed (and in available literature) nevertheless agree that it is essential for engineers to have a good grounding in mathematics including general logic and problem solving. Many important innovations may well be attributed to engineers who did use mathematics directly. One eminent applied scientist, a Fellow of the Academy of Technological Sciences and Engineering, wrote,

“for all of the 4 National Research Priorities (Environment, Health, Wealth and Security) one could argue that Mathematical Modelling and Maths/Stats tools add considerable value (and indeed are even arguably indispensable) as a basis for decision making, (possibly also for innovation).....And what about the old fashioned idea that the discipline of math modelling forces one to think clearly: (one) can't get a math formulation otherwise.”

There is particular concern from all quarters that engineering students enter the workplace lacking confidence in the use of statistical modelling and risk analysis. Confidence with these mathematical prediction techniques would have a number of economic benefits for employers (Trevelyan, 2007).

Twenty years ago it was uncommon for an engineering degree program to contain as few as four semesters of mathematics. Now the average number of single semester mathematics subjects taken is 3.5. In most cases, this has meant that some mathematics topics have had to be removed from the compulsory part of the curriculum. However, there is widespread disagreement among professional engineers and among engineering educators on which mathematics and statistics topics should be included. This implies that some engineers have to be disappointed by the mathematics curriculum. This is a current problem for some mathematics departments. In some instances when two semesters of second year mathematics have been reduced to one, the content of the new single subject contains 60% or more of that which used to be covered in two subjects. The students are having trouble keeping pace, and they are becoming disaffected. The problem is minimised in those institutions where the engineering departments and the mathematics department have a formal joint committee that communicates openly and decides on a compromise in the mathematics curriculum on which both parties agree.

It is fair to say that in general, only a very small number of single degree BE students are extensively trained in mathematics and statistics. These are students who choose mainly mathematics subjects as their optional subjects. A small but important number of students are enrolling in double degrees majoring in a mathematical or physical science. In at least three universities, the number of such students is sufficient to make up a separate class at second and third year levels. They are very well trained, and well suited for research but they pay a penalty of at least one year's extra enrolment.

There is an increasing expectation among engineering educators that mathematics should be taught in the context of good engineering examples. Many caution that moving to teaching all mathematics in context limits students' ability to transfer it to different contexts, which is particularly problematic in large multi-specialisation/discipline lectures and classes. There are reports of effective subjects that teach mathematics out-of-context first, then provide students with in-context examples relevant to their specialisations, which are then covered in tutorials. Wood (2003) actually reports student dissatisfaction with a subject that taught mathematics through three in-context examples over one semester, with students struggling to transfer the mathematics to exam questions in other contexts. Britton *et al* (2005, 2007) also reported that less mathematically able students struggled applying mathematics out of the context in which they were taught.

However, many mathematical techniques have been developed in response to problems in engineering, and mathematics educators should make reference to that context. In lectures delivered by mathematics departments, this is rarely happening. Mathematics departments continue to shrink, as they have done over the past ten years. According to our own census, in February 2008 there are 36 fewer academic mathematicians than there were in February 2007, a decrease of close to one per department. Many Australian mathematics departments employ applied mathematicians who have made contributions to engineering science; in many cases, due to scheduling difficulties, these people are not teaching engineering students.

At this time of relatively high economic prosperity, largely driven by high international demand for mineral exports, there is a high demand for qualified engineers. The Engineering Faculties hope to enrol around 10,000 new domestic students annually (Australian Council of Engineering Deans, 2008). Each year only about 60,000 students complete Year 12 intermediate and advanced mathematics; many of these enter fields other than engineering. In most Group of Eight Universities, the majority of domestic students entering undergraduate engineering programs have indeed completed Year 12 advanced mathematics but a significant minority of them have not. In most of the other 24 institutions awarding engineering degrees, this situation is reversed; students with a background of advanced mathematics are in the minority. Many undergraduate engineering program descriptions state that they assume students' knowledge of intermediate Year 12 mathematics. However in many cases, they admit students with elementary Year 12 mathematics (still including a small amount of calculus). In some instances they admit students with no experience of calculus. There is also growth in admissions through alternative pathways such as TAFE. There has been strong growth in the intake of students from Asia. Usually, they are admitted only after their scholastic record has been well scrutinised, sometimes after they have completed a preliminary year of foundation studies. Their performance at university is usually at least as strong as that of the group of domestic entrants except when their English language skills are insufficient.

Students who have avoided mathematics at secondary school are likely to avoid the subject at university. Some mathematics lecturers demonstrated that most of their failures in first year mathematics were attributed to students who miss the majority of scheduled classes. Some of the non-attendances and incomplete assignments are attributable to competing demands from employers. Unlike 20 years ago, the majority of students are in paid outside employment. Even if they do regularly attend, students with weaker mathematics backgrounds are less likely to succeed, not only in future mathematics but also in mathematically based engineering subjects such as engineering mechanics. There is a high correlation between grades in mathematics and in mechanics. In fact, systematic studies in USA show that success in mathematics is a strong predictor for future success even in more remote subjects (Adelman, 1999, Rosenbaum, 2001).

Universities are responding to the greater diversity in students' mathematics backgrounds in various ways, some by offering two streams of first year mathematics, some by offering pre-entry bridging courses, some by offering parallel developmental courses during semesters, many by extending hours of student drop-in help centres or email inquiry lines. Some are yet to decide. The problem demands attention. A small number of institutions have resolved to incorporate some of the mathematics learning within integrated problem-based learning groups. The learnt mathematics is then demonstrated in student project reports rather than in examination scripts.

One problem then is that the instructors have less control over what mathematical topics are encountered. In the small number of subjects wherein this has been tried, there is reported a higher level of student satisfaction. Exciting a student in any area of engineering mathematics might be preferable to steering that student through a succession of 50% passes on examinations. Objective comparisons of the long-term learning from PBL and from more formal classes is difficult. The relative merits of each are likely to be debated for a number of years.

The change in the student body, both in expectation and widening ability levels is well documented and there is increasing recognition of the need for additional student support in the transition from secondary to tertiary education. Many students struggle with the change in teaching style of mathematics in tertiary education and often are unable to keep up with the pace without one-on-one support. There is some discussion of this change in teaching styles (particularly in mathematics) as a contributor to attrition rates in first year tertiary education, as students are able to miss lectures without being challenged and possibly find themselves unable to catch up. The introduction of the FYE at CUT saw a notable drop in the attrition rate of students and increased satisfaction with mathematics.

Some universities have enormous enrolments of engineering students in first and second year mathematics. Some lecture classes have as many as 600 students in one theatre. In such large classes, it is all too easy for students to hide and not to ask for assistance when they really need it. Four universities have virtually solved this problem by running compulsory web-based quizzes. A lecturer is automatically alerted when a student does not complete the tests. The student can then be called in for counselling well before the deadline for dropping subjects. They have found that student performances have lifted since they introduced the system.

The increasing reductions of mathematics and statistics staff at Australian universities are leading to larger lecture groups of less well mathematically prepared students. The increased support required by these students for them to succeed is leaving mathematics and statistics staff over-stretched and unable to provide the small group contact time or one-on-one support previously available to students. Diagnostic software that allows (or enforces) the practice of mathematics as discussed in section 4.3.3, without added marking pressure on already overstretched mathematics and statistics staff, may provide an effective solution for many institutions, particularly in identifying those students who are struggling or unable to keep up. Some propose a co-ordinated approach to the introduction of CAA software in Australia (Hadgraft, 2007, Keady *et al*, 2006), with some supporting a national database which may provide a bank of questions which educators may draw on for student assessment.

From our observations, mathematics departments that are achieving high levels of satisfaction among engineering academics and their students, are those that have a formal mechanism for regular consultation and joint course planning, offer regular drop-in assistance for students outside of classes, and are flexible in designing assessment tasks for students with special needs.

6. Vision Statement

Our vision is for an engineering mathematics education system that:

- universally accepts the importance of some formal education in mathematics,
- caps the diversity of mathematics preparedness of incoming students at the current level,
- has some recognised pathway for a group of students to emerge as 4-year engineering graduates with exceptionally strong training in mathematics,
- has joint ownership of curriculum design shared by mathematics and engineering disciplines,
- has a more open attitude to shared multidisciplinary teaching,
- has an open national discussion forum on ways of better engaging students, including ready access to helpful CAA systems,
- involves mathematics academics with a good understanding of the engineering context, and
- has a flexible attitude in choice of assessment modes for students who have previously lacked success in mathematics.

7. Recommendations

The recommendations made here, build on innovations that we have identified around the country. They provide strategies to address the considerable challenges that emerge from the Section 5 The Current State of Engineering Mathematics.

The challenges are to achieve:

- effective course design and delivery for an increasingly diverse body of students,
- agreement in the selection of a limited number of topics in the curriculum,
- efficient monitoring of progress of, and formative feedback to, large classes,
- improved motivation of students by better relating mathematics to the engineering context,
- effective assistance to students with various academic backgrounds, and
- recognition by the mathematics and engineering departments that these challenges are shared as a national problem, requiring sharing of ideas, joint development of learning and assessment materials, and joint strategies.

7.1 Recommendation 1: assumed knowledge

That engineering programs should continue to state that students will be assumed to have knowledge of material covered in Year 12 Intermediate Mathematics, including some calculus. For those students entering without that knowledge, an additional developmental subject must precede the normal mathematics subjects.

- The widening diversity of incoming students' mathematical preparedness is stretching the resources of the teaching departments (see Sections 4.2, 4.3.1, 4.3.4). This recommendation would help to cap that diversity at its current level; otherwise the quality of the programs will be compromised.
- If this level of assumed knowledge were removed, then it would likely lead to a further reduction of enrolments in Year 12 Intermediate Mathematics (see Sec 4.1.2), decreasing the pool of mathematically well-trained school leavers from which Engineering Faculties traditionally draw students.
- For a definition of intermediate Year 12 mathematics and a state-by-state comparison, we refer to Barrington and Brown (2005).

Action: Associate Deans of Engineering (Teaching and Learning) or their equivalent to ensure that statements of assumed mathematical knowledge are included on all degree program descriptions, on web pages and in printed information. If these requirements are expected to be waived, then expectations of backgrounds of non-compliant students and their number should be conveyed to the mathematics department.

Action: Mathematics Heads to delegate to appropriate lecturers, the task of appropriate course design for a developmental subject, if students without assumed knowledge are expected to be enrolled.

7.2 Recommendation 2: designated quantitative stream

That in 4-year BE programs with a first-year intake of 140 or more, 15% or more of the places be reserved for a designated quantitative stream in which students must take at least 5 subjects of mathematics, statistics, theoretical computer science, quantitative finance and theoretical physics.

- It is important for practical problem-solving capability, for innovative design and for underpinning engineering science research that we maintain at least a small output of engineering graduates who are extensively trained in mathematical modelling and mathematical methods (see Sections 2.2, 5).
- It is intended that students in the quantitative stream will major in one of the existing specialisations of engineering. However, students in the designated quantitative stream could be given a suggested outline of their program, with suggested subjects in quantitative sciences and quantitative engineering.
- The indicative figures of 15% of an intake of 140 or more (typical size of first year lecture classes, Section 4.5) imply a special class of 20 or more, which should be feasible at second or third year level. The additional subjects may be pre-existing for mathematics or science students. They may be shared among institutions making use of available technology such as video conferencing and access grid rooms.
- The inclusion of 5 subjects devoted to mathematical material is significantly more than the average of 3.5 in current programs (Section 4.5) and it is comparable to the minimum engineering mathematics content for BE degrees in US research universities (Sections 4.7.2, 4.8).

Action: Engineering departments to choose a list of preferred mathematical subject options for quantitative strand of their specialisation.

Action: Engineering Faculties to make an information sheet on the quantitative strand of BE, and to promote it as an important option.

7.3 Recommendation 3: statistics and stochastic modelling

That a single one-semester optional subject in statistics and stochastic modelling be made available to all engineering students who have completed three mathematics courses, if not already included in the syllabus.

- We have identified some unmet demand for training in relevant statistical and probabilistic methods (Sections 4.5, 4.6, 5).
- From our survey, in several universities, a relevant course in statistics and stochastic modelling is already compulsory for electrical, telecommunications and computer engineering students. Such a course should also be made available as an option for other specialisations.

Action: Engineering Faculties to review probability, statistics, stochastic modelling and risk management courses currently available to their students and to convey their impressions to the mathematics departments.

Action: Mathematics Departments to negotiate with Engineering Faculties to modify or design one subject in probability and statistics.

Action: Engineering Faculties to designate on student guides a preferred optional subject in mathematics and statistics,

7.4 Recommendation 4: joint mathematics curriculum committees

That every engineering program has a joint mathematics curriculum committee that is responsible for determining mathematical topics to be covered. The committee should meet at least twice per year and it should have representatives from engineering, mathematics and statistics departments, as well as two students who have recently completed some engineering mathematics subjects.

- Difficult decisions must be made on which mathematical topics to omit for those students taking the minimum number of mathematics subjects. It is essential to have good communication between the mathematics departments and the engineering departments so that curriculum decisions can be jointly owned (Section 4.6).
- In order to enhance communication, it is advisable to include from each of the serving and the client department, a senior academic with long experience in the needs of the subject at that institution.

Action: Engineering Faculties and Mathematics Departments to write guidelines (including purpose, frequency of meetings, composition and means of selecting members) and form joint curriculum committees.

7.5 Recommendation 5: collaborative teaching

That universities modify their internal financial allocation system so that no budgetary unit is penalised for taking part in genuine multidisciplinary collaborative teaching.

- There is a demand from engineering staff and students to relate the mathematics material more to the engineering context (Sections 4.3.2, 4.3.3). This will require staff expertise in both mathematics and engineering. One way to achieve this is through multidisciplinary team teaching.
- Collaborative teaching activity is hampered by inter-faculty rivalries fuelled by differential funding formulae based on weighted equivalent full-time student units (Sections 4.5, 4.6). This is an unfortunate sub-optimal use of human resources: some engineers are well trained in mathematics and some mathematicians are expert in topics taught in engineering (e.g. dynamics, fluid mechanics, signal processing).
- For example it should be possible to modify a budgetary system so that if two mathematics staff teach parts of two engineering courses and two engineers teach parts of mathematics courses, then neither of the collaborating faculties is penalised.

Action: Pro Vice-Chancellors (Teaching and Learning) or their equivalent to investigate ways of removing budgetary barriers to collaborative teaching. Perhaps make use of the extra mathematics income from the Discipline Funding Model announced in Federal Budget of 2007.

7.6 Recommendation 6: engineering mathematics staff expertise

That mathematics departments in BE or ME-awarding institutions should identify which of their staff, if any, have knowledge of engineering applications. If this expertise is lacking, some future academic job advertisements should say that ability to teach mathematics in engineering contexts would be an advantage.

- Mathematics staff have varying levels of knowledge of engineering applications. From site visits, we have learnt that engineering mathematics courses are not always being taught by the most appropriate staff.
- The engineering disciplines are arguably the most important clients of mathematics departments and teaching engineering students should be recognised in a tangible manner as part of their core business.

Action: Heads of Mathematics to designate on department home page, those staff with interests in engineering mathematics. If there are none, the issue should be discussed in Department meetings with a view to making such expertise a priority in future hiring.

7.7 Recommendation 7: on-line formative assessment

That mathematics departments, assisted by the Australian Mathematical Sciences Institute and Australian Association for Engineering Education, investigate the introduction of automated systems of test generation, automatic marking and feedback, so that they can run compulsory on-line quizzes during semesters for large engineering mathematics classes.

- We have found that large classes are performing better after the introduction of compulsory on-line automatically generated quizzes with rapid feedback (Sections 4.3.3, 4.4).
- Further investigation needs to be made on preferred computer software platforms so that information on this material can be better disseminated.

Action: AMSI and AAEE to form a project team to investigate compatibility of CAA software, to find the best means of providing a central item bank.

Action: AMSI to negotiate with HELM to subscribe to on-line testing service.

7.8 Recommendation 8: collaborative item bank

That engineering and mathematics teaching departments collaborate to provide a central bank of good examples of formative test questions, computer laboratory projects and curriculum resources.

- Site visits demonstrated that much effort is being duplicated in designing formative tests, assignment questions and computer laboratory projects that clearly test learning objectives, relate to engineering needs and motivate inquiry (Sections 4.4, 4.7.1).
- Of particular interest is a common need to find good examples of mathematics questions in the engineering context (Sections 4.3.2, 4.3.3).
- Also of particular interest is a common need to write questions in a format that can easily be implemented on CAL platforms (Sections 4.3.3, 4.4).

Action: AMSI and AAEE to form a project team to solicit contributions of exemplary test items and curriculum resources and to set up a program of continual improvement of the item bank.

Action: AMSI to negotiate with HELM to subscribe to obtain curriculum support booklets in subset bundles.

7.9 Recommendation 9: student help centres

That Engineering Faculties designate at least 4 common hours per week of class free time spread over 3 or more days and that servicing mathematics departments provide staff or senior students in student help centres at those times.

- Education of diverse student groups seems to be working better when student drop-in centres are staffed by mathematics instructors and/or senior peers at times when students are free (Sections 4.3.1, 4.7.1). Designating class-free hours for students is a scheduling problem that has to be negotiated among many departments.
- In addition, peer-facilitated support sessions for selected subjects have been shown to improve performance (Section 4.3.1).

Action: Associate Deans of Engineering (Teaching and Learning) or their equivalent to schedule 4 hours of formal class-free time covering at least 3 days, for each of first year and second year engineering programs.

Action: Associate Deans responsible for mathematics and engineering or their equivalent to investigate funding student drop-in centres.

7.10 Recommendation 10: boosting senior secondary school mathematics

That able students be more strongly encouraged to progress to subjects comparable to Intermediate Year 12 Mathematics of New South Wales and Victoria.

- Many correspondents have made it clear that improving the pipeline to engineering mathematics depends on reversing the decline of enrolments in Intermediate and Advanced Year 12 Mathematics in schools (Section 4.1.2, 4.2).
- An information campaign is necessary on the breadth of employment opportunities opened up by studying mathematics

Action: AMSI to confer with Engineers Australia to construct suitable careers guidance materials for schools and to plan broader publicity campaign.

8. Linkages

An ongoing linkage has been formed between AMSI and AAEE to improve mathematics education for engineers. We are working together to reinvigorate an existing AAEE interest group in mathematical education. This interest group will be broadened to include members of the mathematics community who are not members of AAEE. We have received an encouraging response from the President of the Engineering Mathematics Group, an interest group of the Australian Mathematical Society, to coordinate the mathematicians' participation.

The intention is for the interest group to set up an edited special interest web page on mathematical education in practical contexts. This page will have a discussion board as well as a collection of regularly contributed longer articles (e.g. one per month).

AAEE has approached the Australasian Journal of Engineering Education to run a special guest-edited issue on engineering contexts for teaching mathematics.

Another important linkage has been formed with the HELM network (Helping Engineers Learn Mathematics) for which the main node is based at Loughborough University in the UK. HELM has produced an extremely useful set of topic study guides that covers the whole range of mathematical topics encountered in engineering education.

9. Further Development

It is clear that many Australian mathematics departments face similar problems in engaging engineering students in mathematics. Our study has taken stock of the many curricular and pedagogical adaptations and innovations that are being introduced independently at various institutions. A more coordinated effort in educational development is needed at the national level in order for the mathematics discipline to maintain a good relationship with the engineering profession. When demonstrable improvements are made locally in engineering mathematics, there is every reason to spread the news.

We have found that mathematics departments are not often using modern applications as contexts in their teaching. As explained in Section 9, we are taking steps to inform our colleagues of some stimulating contexts that may be useful for this purpose. This will require further investigation and coordination.

In future we would like to provide a collection of examples of short topic test items that could be implemented on CAA software. We need to investigate the issue further before we can present this material in a format that is easily implemented on many software platforms. Alternatively we need to present the material so that it is aligned with a smaller number of preferred platforms. Our survey of which software packages are being used, has helped inform us. Some mathematical software is used more universally than we had anticipated, giving us scope for centralised development of teaching materials. However, more research and experimentation will need to be done before we can resolve this issue.

We would like to maintain a repository of top quality student self-help guides that will enable lecturers to avoid duplication of effort. The HELM topic guides are excellent and we would like to adapt them for the Australian context. We would also like to provide a student's abbreviated introduction to MATLAB and to other commonly used software packages. At this time, many universities are duplicating this effort and the guides are not being compared so that the best version can be synthesized.

We would also like to consult with experts to produce guides on:

- how to engage students in large classes
- how to provide automatic diagnostic feedback
- how to provide flexible assessment systems to give more students a chance to demonstrate their learning
- how to write computer laboratory exercises for engineering students
- how to steer problem-based-learning groups
- how to design a multidisciplinary capstone course.

These questions have a research component because their answers are still being debated.

Bibliography

- Adelman, C. (1999). *Answers in the Toolbox: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment*, U.S. Dept. of Education.
- Aminifar, E., Porter, A., Caladine, R. and Nelson, M. I. (2007). Creating mathematical learning resources – combining audio and visual components. *Proceedings of the 7th Biennial Engineering Mathematics and Applications Conference* (pp. 934-955). Melbourne, Australia.
- Anderson, M., Bloom, L., Mueller, U. and Pedler, P. (2000). Enhancing the Teaching of Engineering Differential Equations with Scientific Notebook. *International Journal of Engineering Education*, Vol. 16, No. 1, pp 73-79.
- Australian Council of Engineering Deans (2008). *Addressing the Supply and Quality of Engineering Graduates for the New Century*. Carrick Institute for Learning and Teaching in Higher Education. Available online at: http://www.carrickinstitute.edu.au/carrick/webdav/users/siteadmin/public/Grants_DBLprojec_engineeringquality_project%20report_25march08.pdf
- Australian Vice-Chancellor's Committee (2005). *Key Statistics on Higher Education*. Available online at: <http://www.universitiesaustralia.edu.au/documents/publications/stats/2005Edition.pdf>
- Avitabile, P., McKelliget, J., and VanZandt, T. (2005). Interweaving numerical processing techniques in multise semester projects. *Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition*, Oregon, USA. Available online at: <http://dynsys.uml.edu/technicalpapers.htm#2005%20-%20ASEE%20Conference>
- Baafi, E. and Boyd, M. (2001). *Developing a series of online tutorials for a range of linked, discrete topics*. Centre for Educational Development and Interactive Resources: University of Wollongong. Available online at: http://cedir.uow.edu.au/CEDIR/programs/samples/ld_linkedtopicsflash.html
- Barrington, F. and Brown, P. (2005). *Comparison of Year 12 mathematics subjects in Australia 2004-2005*. Available online at: www.amsi.org.au/pdfs/comp_y12pretertiary_au_200404-05.pdf
- Barrington, F. (2006). *Participation in Year 12 Mathematics across Australia 1995-2004*. Available online at: www.amsi.org.au/pdfs/Participation_in_Yr12_Maths.pdf
- Barry, S. I. and Webb, T. (2006). Multi-disciplinary Approach to Teaching Mathematics to Engineers using Matlab. *Proceedings of the 7th Biennial Engineering Mathematics and Applications Conference*. Melbourne, Australia.
- Barry and Healy (2007). *Preliminary Study on Undergraduate Engineering Mathematics in Australia*. School of Physical, Environmental and Mathematical Sciences, University of New South Wales, Australian Defence Force Academy
- Barry, S.I. and Chapman, J. (2007). Predicting University Performance. *Proceedings of the 8th Biennial Engineering Mathematics and Applications Conference*. Tasmania, Australia.
- Barry, S. and Davis, S. (2008). *Essential Mathematics Skills* (2nd edition), UNSW Press, Australia.
- Belward, S. R., Mullamphy, D. F. T., Read, W. W. and Sneddon, G. E. (2005). Preparation of students for tertiary studies requiring mathematics. *Proceedings of the 7th Biennial Engineering Mathematics and Applications Conference*. Melbourne, Australia.
- Blake, B. and Reed, M. (2004). *31L -32L Coursepack 2004-2005: Laboratory Calculus*. Department of Mathematics, Duke University.
- Bloom, W. (2007). Motivating the Teaching of Mathematics to Engineering Students using Scientific Notebook. *Proceedings of the 18th Annual Australian Association of Engineering Education Conference*, Melbourne, Australia. Available online at: <http://www.amsi.org.au/Carrick/14%20-%20Bloom.pdf>
- Blyth, W. F., Clarke, D. and Labovic, A. (2005). Video Analysis to Understand e-learning of Vector Calculus. *Proceedings of the 7th Biennial Engineering Mathematics and Applications Conference*.

Melbourne, Australia.

Blyth, W. F. (2007a). Lecture Interactive Computer Aided Assessment Using Maple in a Finite Element Method Course. *Proceedings of 8th Biennial the Engineering Mathematics and Applications Conference*. Hobart, Australia.

Blyth, W. F. (2007b). Computer Aided Assessment Using Maple in a FEM Course. *Proceedings of the National Symposium on Mathematics for 21st Century Engineering Students*, Melbourne, Australia. Available online at: http://www.amsi.org.au/carrick_seminar_program.php

Blockley, D. and N. Woodman, N. (2003). The changing relationship: civil/structural engineers and Maths. (In LTSN MathsTEAM Project. *Maths for Engineering and Science*). Available online at: http://mathstore.ac.uk/mathsteam/packs/engineering__science.pdf

Boland, J. (2002). The Mathematics Bridging Course at the University of South Australia. *Proceedings of the Second International Conference on the Teaching of Mathematics*. Crete, Greece. Available online at: <http://www.math.uoc.gr/~ictm2/Proceedings/pap213.pdf>

Boyer Commission on Educating Undergraduates in the Research University (2007). *Reinventing Undergraduate Education: A Blueprint for America's Research Universities*. Carnegie Foundation. Available online at: <http://naples.cc.sunysb.edu/pres/boyer.nsf>

Brenner, A., Shacham, M. and Cutlip, M. B. (2005). Applications of mathematical software packages for modelling and simulations in environmental engineering education. *Environmental Modelling & Software*, Vol. 20, No. 10, pp 1307-1313

Britton, S., Daners, D. and Stewart, M. (2007). A self-assessment test for incoming students. *International Journal of Mathematical Education in Science and Technology*. Vol. 28, No. 7, pp 861-868.

Britton, S., New, P., Sharma, M., and Yardley, D. (2005). A case study of the transfer of mathematical skills by university students. *International Journal of Mathematical Education in Science and Technology*, Vol. 30, No. 1, pp 1-13.

Campbell, D., Boles, W, Murray, M., Iyer, M., Hargreaves, D. and Keir, A. (2007). Balancing Pedagogy and Student Experience in First Year Engineering Courses. *Proceedings of the 3rd International Conceive Design Implement Operate (CDIO) Conference*, Massachusetts, USA.

Colgan, L. (2000). MATLAB in First-year Engineering Mathematics. *International Journal of Mathematics Education in Science and Technology*, Vol. 31, No. 1, pp 15-25

Cretchley, P. (1999). An Argument for More Diversity in Early Undergraduate Mathematics Assessment. *Proceedings of the 2nd Australian Symposium on Modern Undergraduate Mathematics*. Queensland, Australia. Available online at: <http://www.sci.usq.edu.au/staff/spunde/delta99/Papers/cretchl.pdf>

Croft, A. and Ward, J. (2001). A modern and interactive approach to learning engineering Mathematics. *British Journal of Educational Technology*, Vol. 32, No 2, pp. 195-207.

Curtin University of Technology Engineering Faculty (undated). *Engineering Foundation Year: An Overview*.

Cuthbert, R. and MacGillivray, H. (2003). Investigating weaknesses in the underpinning mathematical confidence of first year engineering students. *Proceedings of the 14th Annual Australian Association of Engineering Education Conference*. Melbourne, Australia.

Cuthburt, R. H. and MacGillivray, H. (2007). Investigating Completion Rates of Engineering Students. *Proceedings of the 6th Southern Hemisphere Conference on Mathematics and Statistics Teaching and Learning*. Calafate, Argentina.

De Hosson, G. (2007). *PASS: MATH 141 Report – Autumn 2007*. School of Mathematics and Applied Statistics, University of Wollongong (internal document)

- Department of Education Science and Training (2007). *2006 Full Year Higher Education Student Data*. Available online at: http://www.dest.gov.au/NR/rdonlyres/ECADEDDB-C358-4B97-9244-7A74D9974061/18815/2006FullYearStudentdata_shortanalysis3.pdf
- Department of Education, Science and Training (2002). *Striving for quality: learning, teaching and scholarship*. Available online at: http://www.backingaustraliasfuture.gov.au/publications/striving_for_quality/pdf/quality.pdf
- Dobson, I. R. (2007). Sustaining Science: University Science in the Twenty-First Century. A Study commissioned by the Australian Council of Deans of Science. Available online at: <http://www.acds.edu.au/>
- Donea, A. and Lun, T. (2007). Cherchez la physique: the physics behind (engineering) mathematics. *Proceedings of the National Symposium on Mathematics for 21st Century Engineering Students*. Melbourne, Australia. Available online at: http://www.amsi.org.au/carrick_seminar_program.php
- Dorfler, W. (2003). Mathematics Education: Context and People, Relation and Difference. *Educational Studies in Mathematics*, Vol. 54, No. 2/3, pp147-170.
- Duran, M. J., Gallardo, S., Toral, S. L., Martinez-Torres, R. and Barrero, F. J. (2007). A learning methodology using Matlab/Simulink for undergraduate electrical engineering courses attending to learner satisfaction outcomes. *International Journal of Technology and Design Education*, Vol. 17, No. 1, pp 55-73.
- Engineers Australia (2006a). *The Engineering Profession A Statistical Overview*. Available online at: http://www.engineersaustralia.org.au/shadomx/apps/fms/fmsdownload.cfm?file_uuid=8942979D-9162-240C-EA11-AE840DBB2CEC&siteName=ieaust
- Engineers Australia (2006b). *Accreditation Criteria Guidelines*. Available online at: http://www.engineersaustralia.org.au/shadomx/apps/fms/fmsdownload.cfm?file_uuid=0B19D0FF-0BC5-BAC1-DB36-6FB8599DDE67&siteName=ieaust
- Engineers Australia (2007). *Technically Speaking - Queensland. Confronting the challenges facing science, engineering, technology and mathematics education and promotion*. Available online at: http://www.engineersaustralia.org.au/shadomx/apps/fms/fmsdownload.cfm?file_uuid=AAEE5C66-AA75-349D-73C6-8B90788F1640&siteName=ieaust
- Felder, R. and Silverman, L. (1988). Learning and Teaching Styles in Engineering Education. *Engineering Education*, Vol. 78, No. 7, pp 674-681.
- Felder, R. (2002). *Author's Preface: Learning and Teaching Styles in Engineering Education*. Available online at: <http://www4.ncsu.edu/unity/lockers/users/f/felder/public/Papers/LS-1988.pdf>
- Fernandez, G. and Fitz-Gerald, G. (2004a). Towards an automated web tutor environment. In E. McKay (Ed.), *Proceedings of the 12th International Conference on Computers in Education*. Melbourne, Australia.
- Fernandez, G. and Fitz-Gerald, G. (2004b). A Bold Mathematics Course to Support a New Civil Infrastructure Program (RMIT's Experience). *Proceedings of the 9th Asian Technology Conference in Mathematics*. Singapore.
- Field, B., Weir, J. and Burvill, C. (2003). An Electronic Tutor for Mechanical Engineering. *Proceedings of the 14th Annual Australian Association of Engineering Education Conference*. Melbourne, Australia.
- Fitz-Gerald, G.F. and Healy, W.P. (1994). Enlivening the Mathematics Curriculum with Maple. In Lopez (ed) *Maple V: Mathematics and its Applications (Proceedings of the Maple Summer Workshop, New York)*, Birkhauser, Boston.
- Hadgraft, R. (2007). It's time for a co-ordinated approach to computer-aided learning and assessment, *Proceedings of 18th Annual Australian Association of Engineering Education Conference*, Melbourne, Australia.

- James, R., Bexley, E., Devlin, M. and Marginson, S. (2007). *Australian University Student Finances 2006*. Available online at: <http://www.universitiesaustralia.edu.au/documents/publications/policy/survey/AUSF-Final-Report-2006.pdf>
- Jayasuriya, K. and Evans, G. (2007). Journeys in problem-based learning during the first year in Engineering. *Proceedings of the 18th Conference of the Australasian Association for Engineering Education*. Melbourne, Australia.
- Johnson, D. W., Johnson, R. T. and Stanne, M. B. (2000). *Comparative Learning Methods: A Meta-Analysis*. Available online at: <http://www.co-operation.org/pages/cl-methods.html>
- Judd, K. (1996). *Teaching Intermediate Calculus by Computer*. Available online at: <https://calmaeth.maths.uwa.edu.au/doc/reports/report1996.html>
- Keady, G., Fitz-Gerald, G., Gamble, G and Sangwin, C. (2006). Computer Aided Assessment in Mathematical Sciences. *Proceedings of the UniServe Science Conference*. Sydney, Australia.
- Kent, P and Noss, R. (2000). The visibility of models: using technology as a bridge between mathematics and engineering. *International Journal of Mathematical Education in Science and Technology*, Vol. 31, No. 1, pp. 61-69.
- Kent, P and Noss, R. (2002a). *The Mathematical Components of Engineering Expertise*. School of Mathematics, Science and Technology, Institute of Education, University of London. Available online at: <http://www.ioe.ac.uk/rnoss/MCEE/MCEE-End-of-Award-report.pdf>.
- Kent, P and Noss, R. (2002b). The mathematical components of engineering expertise: the relationship between doing and understanding mathematics. *Proceedings of the Institution of Electrical Engineers Second Annual Symposium on Engineering Education*. London, United Kingdom. Available online at: <http://www.ioe.ac.uk/rnoss/MCEE/Kent-Noss-EE2002-preprint.pdf>.
- King, R. (2007). Mathematics for Engineers: observations from the Review of Engineering Education. *Proceedings of National Symposium on Mathematics Education for 21st Century Engineering Students*. Melbourne, Australia. Available online at: http://www.amsi.org.au/carrick_seminar_program.php
- Krause, K., Hartley, R., James, R. and McInnis, C. (2005). *The First Year Experience in Australian Universities: Findings from a Decade of National Studies*. Available online at: http://www.dest.gov.au/sectors/higher_education/publications_resources/profiles/first_year_experience.htm#authors
- Lesik, S. A. (2006). Do Developmental Mathematics Programs have a Causal Impact on Student Retention? An Application of Discrete-Time Survival and Regression-Discontinuity. *Research in Higher Education*, Vol. 48, No. 5, pp 583-608.
- Lewis, D., O'Brien, M. J., Rogan, S. and Shorten, B. (2005). *Do Students Benefit From Supplemental Instruction?* Social Science Research Network. Available online at: <http://ssrn.com/abstract=811846>
- Loch, B. and Donovan, D. (2006). Progressive Teaching of Mathematics with Tablet Technology. *E-Journal of Instructional Science and Technology*. Available online at: http://www.usq.edu.au/electpub/e-jist/docs/vol9_no2/papers/current_practice/loch_donovan.htm
- Loch, B. and Donovan, D. (2007). Progressive Teaching of Undergraduate Engineering Mathematics with Tablet Technology. *Proceedings of National Symposium on Mathematics Education for 21st Century Engineering Students*. Melbourne, Australia. Available online at: http://www.amsi.org.au/carrick_seminar_program.php
- Lopez, A. (2007). *Mathematics for 21st Century Engineering Students: Literature Review*. Part of a Carrick Institute for Teaching and Learning funded review of mathematics for 21st century engineering students. Available online at: www.amsi.org.au/carrick_project_documents/literature_review.pdf
- Lui, D. K., Huang, S. D. and Brown, T. A. (2007). Supporting Teaching and Learning of Optimisation Algorithms with Visualisation Techniques, *Proceedings of 2007 Australian Association of Engineering Education Conference*, Melbourne.

LTSN MathsTEAM Project (2003a). *Maths for Engineering and Science*. Available online at: http://mathstore.ac.uk/mathsteam/packs/engineering__science.pdf

LTSN MathsTEAM Project (2003b). *Diagnostic Testing for Mathematics*. Available online at: http://mathstore.ac.uk/mathsteam/packs/diagnostic_test.pdf

LTSN MathsTEAM Project (2003c). *Maths Support for Students*. Available online at: http://mathstore.ac.uk/mathsteam/packs/student_support.pdf

McCrindle, M. (2007). *New Generations, new trends: Snapshots of Australia's changing demographics, generations, & population shifts*. Available online at: http://mccrindle.com.au/wp_pdf/BridgingTheGap_Employers.pdf

McKenzie, K and Schweitzer, R. (2001). Who Succeeds at University? Factors Predicting Academic Performance in First Year Australian University Students. *Higher Education Research and Development*, Vol 20. No. 1, pp.21-33.

MacGillivray, H. (2007). The multi-layered challenges of teaching statistics in engineering courses. *Proceedings of the National Symposium on Mathematics for 21st Century Engineering Students*. Melbourne, Australia. Available online at: http://www.amsi.org.au/carrick_seminar_program.php

Michalewicz, Z. and Michalewicz, M. (2007). Puzzle-Based Learning. *Proceedings of the 18th Conference of the Australasian Association for Engineering Education*, Melbourne, Australia.

Mills, J. E. and Treagust, D. F. (2003). Engineering Education – Is Problem-Based or Project-Based Learning the answer? *The Australasian Journal of Engineering Education*, online publication 2003-04. Available online at: http://www.aeee.com.au/journal/2003/mills_treagust03.pdf

Morgenroth, E., Arvin, E. and Vanrolleghem, P. (2002). The use of mathematical models in teaching wastewater treatment engineering. *Water Science and Technology*, Vol. 45 No. 6, pp 229-233.

Mtenga, P. V. and Spainhour, L. K. (2000). Applications of mathematical software packages in structural engineering education and practice. *Journal of Computing in Civil Engineering*, Vol. 14, No. 4, pp 273-278.

Mulligan, D and Kirkpatrick, A. (2000). How much do they understand? Lectures, students and comprehension. *Higher Education Research and Development*, Vol. 19, No. 3, pp 311-335.

Mullamphy, D., Read, W. and Belward, S. (2007). How well are we educating our mathematics students? *Proceedings of the 8th Biennial Engineering Mathematics and Applications Conference*, Tasmania, Australia.

Naimark, A. (2002). Applications, MATLAB and linear algebra as a unifying vehicle for the engineering-oriented syllabus. *European Journal of Engineering Education*, Vol 27, No. 4, pp 409-424.

National Committee for the Mathematical Sciences of the Australian Academy of Science (2006). *Mathematics and Statistics: Critical Skills for Australia's Future*. Available online at: www.review.ms.unimelb.edu.au

Nirmalakhandan, N., Ricketts, C., McShannon, J. and Barrett, S. (2007). Teaching tools to promote active learning: Case study. *Journal of Professional Issues in Engineering Education and Practice*, Volume 133, No. 1, pp 31-37.

O'Brien, M. (2006). *An analysis of the effectiveness of the Peer Assisted Study Sessions (PASS) program at the University of Wollongong: Controlling for self-selection*. School of Economics, The University of Wollongong.

Ostrogonac—Seserko, R., Scott, N. and Bush. M. (2006). Integrated Learning in engineering education at UWA: designed by students for students. *Proceedings of the 4th International Conference on Multimedia and Information and Communication Technologies*. Available online at: www.formatex.org/micte2006/pdf/1274-1278.pdf

- Roberts, A., Sharma, M., Britton, S. and New, P. (2007). An index to measure the ability of first year science students to transfer mathematics. *International Journal of Mathematical Education in Science and Technology*, Vol. 38, No. 4, pp 429-448.
- Rosenbaum, J. (2001). *Beyond College for All: Career Paths for the Forgotten Half*, Russell Sage Foundation.
- Sangwin, C. (2006). *Assessing Elementary Algebra with STACK*. Available online at: <http://www.open.ac.uk/opencetl/resources/details/detail.php?itemId=461cdcef52913>
- Sazhin, S. S. (1998). Teaching Mathematics to Engineering Students. *International Journal of Engineering Education*, Vol. 14, No. 2, pp 145-152.
- SEFI Working Group (2002), *Mathematics for the European Engineer: A curriculum for the twenty-first century*, SEFI Mathematics Working Group, <https://learn.lboro.ac.uk/mwg/core/latest/sefimarch2002.pdf>
- Senate Committee Report (2007). *Quality of school education*. Available online at: http://www.aph.gov.au/Senate/committee/eet_ctte/academic_standards/report/index.htm
- Smith, K. A., Sheppard, S. D., Johnson, D. W. and Johnson, R. T. (2005) Pedagogies of engagement: classroom-based practices. *Journal of Engineering Education*, Vol. 94, No.1, pp 87-101.
- Springer, L., Stanne, M. E. and Donovan, S. S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis, *Review of Educational Research*, vol. 69, No. 1, pp. 21-51.
- Tobin, P. and Lozanovski, C. (2007). Computer Generated Online Formative Tests in Engineering Mathematics. *Proceedings of the National Symposium on Mathematics for 21st Century Engineering Students*, Melbourne, Australia. Available online at: http://www.swin.edu.au/feis/mathematics/staff/clozanovski/AMSI_Math_Ed/AMSI_poster.pdf
- Trevelyan, J. (2007). *Mathematics and Engineering Practice*. Submission to the Project Advisory Committee, Faculty of Engineering, Computing and Mathematics, University of Western Australia. Available online at: <http://www.mech.uwa.edu.au/jpt/pes.html>
- Trevelyan, J. (2008). Random Madeness – Technical Co-ordination in Engineering Practice. *The Engineering Essential*. Available online at: <http://www.ecm.uwa.edu.au/for/alumni/ega/newsletters/2008>
- Vithal, R., Christiansen, I., Skovsmose, O. (1995). Project Work in University Mathematics Education: A Danish Experience: Aalborg University. *Educational Studies in Mathematics*, Vol. 29, No. 2, pp 199-223.
- Waldvogel, J. (2006) Teaching mathematics to engineering students at ETH: Coping with the diversity of engineering studies. *Proceedings of the IDEA League Workshop on Mathematics in Engineering*, Imperial College, London. Available online at: <http://www.math.ethz.ch/~waldvoge/Projects/london.pdf>
- Wilson, T. M. and MacGillivray, H. L. (2007). Counting on the basics: mathematical skills among tertiary entrants. *International Journal of Mathematics Education in Science and Technology*, Vol. 28, No. 1, pp 19-41.
- Worsley, S., Hibberd, K. and Maenhaut, B. (2007). Enhancing the student experience in mathematics through the use of a group project. *Proceedings of the 8th Biennial Engineering Mathematics and Applications Conference*. Hobart, Australia.
- Wood, L. N. and Smith, G. H. (1999). Flexible Assessment. *Proceedings of Delta 1999 The Challenge of Diversity – A Symposium on Undergraduate Mathematics*. Queensland. Available online at: http://www.sci.usq.edu.au/staff/spunde/delta99/Papers/wood_2.pdf
- Wood, L. (2003). *Reflections Across Faculties (Volume 1 & 3)*. Department of Mathematical Sciences, The University of Technology Sydney.

Appendix 1: Mathematical Content for Engineers at International Universities Visited

1.1 Loughborough University

Module Specification:

Here we summarise module specifications for mathematics for mechanical engineering students and Electrical engineering program, from descriptions provided by the Mathematics Education Centre at Loughborough University in January 2008. These specifications are fairly similar to those of the other engineering specialisations.

Mechanical Engineering program

Module	Credit	Pre req
Mathematics for Mechanical Eng (MAA 310)	20	None
Mathematics for Mechanical Eng 3 (MAB 110)	10	MAA310

Mechanical engineering students are required to study 1 module in first year (20 Credits, around 74 contact hours run over 2 semesters) and one module (10 Credit, around 36 contact hours run over 1 semester) in second year.

MAA 310: Content of Mathematics for Mechanical Engineering

Algebra of complex numbers, vectors and matrices.

Solution of systems of linear equations: determinants, matrices and Gauss elimination.

Iterative solution of nonlinear equations (Newton Raphson). Elementary functions including hyperbolic functions.

Ordinary and partial differentiation: techniques and applications including stationary values and errors. Integration: analytical techniques and Simpson's rule, applications (area, mean value, RMS, volumes of revolution).

Ordinary differential equations: first order separable and linear equations, second order linear equations with constant coefficients, applications. Laplace transforms: application to solving ordinary differential equations. Sequences and series: infinite series, convergence, Binomial, Maclaurin and Taylor series.

Method of Teaching, Learning and Assessment

Total student effort for the module: 200 hours on average.

Teaching & Learning: A combination of 48 one-hour lectures and 26 one-hour tutorials* with the remaining time for private study, working on problem sheets and revision for exam.

**Tutorials are where no new material is covered. Either students work through problems and get help from the staff on hand or else the lecturer goes through worked examples.*

Assessment: Coursework: Eight computer-based or in class tests (8x5%= 40%). Summative Examination (60%) (3 hours).

MAB 110: Content of Mathematics for Mechanical Engineering 3

Elementary probability and statistics. Matrix eigenvalue problems, with application to solutions of Ordinary Differential Equations, for example vibrating systems.

Optimisation of functions of several variables, with and without constraints. Fourier series and partial differential equations.

Method of Teaching, Learning and Assessment

Total students effort for the module: 100 hours on average.

Teaching & Learning: A combination of 24 one-hour lectures and 12 one-hour tutorials with the remaining time for private study, working on coursework assignments and problem sheets and revision for exam.

Assessment: Coursework - 2 equal computer based tests (20%). Formal Examination (80%) (2 hours).

Electrical Engineering program

Modules	Credit	Pre- req
Mathematics A (MAA303)	15	None
Mathematics B (MAB303)	20	MAA303

Electrical engineering students are required to study 1 module (15 credit around 60 contact hours run over 2 semesters) in first year and 1 module (20 credit, around 36 contact hours run over 2 semesters) in second year.

Content of MAA303: Mathematics A

Introduction to the module. Overview of basic techniques of arithmetic, algebra, functions, and trigonometry.

Complex numbers: motivation, cartesian form, arithmetic. Argand diagram, Polar form. Multiplication & division in polar form. Exponential form. Euler's relations. De Moivre's theorem and application to solving equations and finding roots.

Application to Phasors. Determinants - evaluation and general properties (up to 3×3). Cramer's rule as an application of determinants. Matrices: basic concepts and algebra (up to 3×3). Square matrices. Adjoint. Inverse matrix. Systems of linear equations: solution by matrix inversion. Solution by Gaussian elimination.

Differentiation: rates of change, gradients of tangents to curves. Definition of derivative from first principles. Techniques: tables, rules, application to gradient of a curve, rates of change. Product, quotient and chain rules. Higher derivatives.

Application to maxima and minima and curve sketching.

Applications to electromagnetism, circuit theory etc. Integration as the limit of a sum. Integration as the reverse of differentiation. Table of integrals. Evaluating definite integrals. Rules: sums, constant multiples, integration by parts. Integration by substitution. Integration using partial fractions. Applications of integration: areas, volumes, mean values, rms.

First order differential equations. Solution by direct integration and by separation of variables. First order linear equations: solution by integrating factor.

Introductory ideas in probability: events, Venn diagrams, compound events, independent events, mutually exclusive events. Laws of probability. Discrete and continuous random variables. Binomial, Poisson and Normal distributions.

Method of Teaching, Learning and Assessment

Total student effort for the module: 150 hours on average

Teaching & Learning: A combination of 54 one-hour lectures** with the remaining time for private study, working on problem sheets and revision for exam.

***The lecturer uses the lecture times to incorporate extra worked examples.*

Assessment: The coursework element comprises 5 computer-based tests ($5 \times 5\% = 25\%$). Four of these will take place in Semester 1. In addition there will be 12 shorter computer based tests delivered throughout the module, of which the best 6 will be used (5%).

Summative Examination (70%) (2 hours).

Content of MAB303: Mathematics B

Vectors: geometric vectors, addition, subtraction, scalar multiplication. Cartesian components of vectors. Scalar and vector products; triple products, applications. Eigenvalue problems.

Revise differential equations: basic ideas, terminology, motivation. Revise first order equations. Solution by direct integration and by separation of variables. First order linear equations: solution by integrating factor. Constant coefficient second order equations. Solution of a system of equations using an eigenvalue analysis.

Taylor's theorem. Taylor series and Maclaurin series. Partial differentiation. Concepts, notation. Small increments and differentials. Taylor's theorem in two variables. Stationary values of a function of two variables. Maxima, minima, saddle points. Least squares line of best fit. Introduction to the Laplace transform. Inversion. Application to the solution of differential equations. Introduction to Fourier series and the Fourier transform. Statistics - types of variable, describing distributions using measures of centre and spread, displaying data graphically, distribution of sample mean, unbiased estimators, hypothesis testing, type 1, 2 errors, comparing differences of sample means. Mathematical description of linear dynamical systems arising in electrical and mechanical engineering using ordinary differential equations, transfer functions, and state-space models - state variables, input and output vectors, state, input, output and transmission matrices. Introduction to transfer functions used to relate the Laplace transforms of the input and output of a linear system.

Method of Teaching, Learning and Assessment

Total student effort for the module: 200 hours on average

Teaching & Learning: A combination of 54 one-hour lectures and 27 one-hour tutorials with the remaining time for private study, working on problem sheets and revision for exam.

Assessment: The coursework element comprises 5 computer-based tests ($5 \times 5\% = 25\%$). Four of these will take place in Semester 1. In addition there will be 12 shorter computer based tests delivered throughout the module, of which the best 6 will be used (5%).

Formal Examination (70%) (2 hours).

1.2 Duke University

Mathematics requirements for Electrical & Mechanical Engineering

Basic Maths Subjects for Electrical and Mechanical Engineering programs
Math 31 or 31L Introductory Calculus 1
Math 32 or 32L Introductory Calculus 2
Math 103 Intermediate Calculus
Math 107 Linear Algebra & Differential Equations
Advanced Maths Subjects for Mechanical Engineering program
Math 108 Ordinary & Partial Differential Equations
Advanced Maths Subjects for Electrical Engineering program
Math 108 Ordinary & Partial Differential Equations
Math 135/SAT 113 Probability & Statistics

Math 31 or 31L: Introductory/Laboratory Calculus 1. Functions, limits, continuity, trigonometric functions, techniques and applications of differentiation, indefinite and definite integrals, the fundamental theorem of calculus.

Prerequisites: -none-

Math 32 or 32L: Introductory/Laboratory Calculus 2.

Math 32: Introductory Calculus 2. Math 32 is a traditional calculus course. Transcendental functions, techniques and applications of integration, indeterminate forms, improper integrals, infinite series.

Math 32L: Laboratory Calculus 2: Second semester of introductory/Laboratory calculus with a laboratory component. Emphasis on laboratory projects, group work, and written reports. Methods of integration, applications of integrals, functions defined by integration, improper integrals, introduction to probability and distributions, infinite series, Taylor polynomials, series solutions of differential equations, systems of differential equations, Fourier series.

Prerequisites: Mathematics 31 or 31L.

Math 103: Intermediate Calculus. Partial differentiation, multiple integrals, and topics in differential and integral vector calculus, including Green's theorem, the divergence theorem, and Stoke's theorem.

Prerequisites: Mathematics 32 or 32L.

Math 135 or STA 113: Probability and Statistics in engineering

Math 135: Probability . Probability models, random variables with discrete and continuous distributions. Independence, joint distributions, conditional distributions. Expectations, functions of random variables, central limit theorem.

STA 113: Statistics. Provides an introduction to probability, independence, conditional independence, Bayes' theorem; discrete and continuous, univariate and multivariate distributions; linear and nonlinear transformations of random variables; classical and Bayesian inference, decision theory, and comparison of hypotheses; and experimental design, statistical quality control, and other applications in engineering.

Prerequisites: Math 103.

Math 107: Linear Algebra and Differential Equations Systems of linear equations, matrix operations, vector spaces, linear transformations, orthogonality, determinants, eigenvalues and eigenvectors, diagonalization, linear differential equations and systems with constant coefficients and applications, computer simulations.

Math 108: Ordinary and Partial Differential Equations First and second order ordinary differential equations with applications, Laplace transforms, series solutions and qualitative behavior, Fourier series, partial differential equations, boundary value problems, Sturm-Liouville theory.

1.3 Delaware University

Mathematics requirements for Electrical & Mechanical Engineering

Maths Subjects Electrical program
Math 241 Analytical Geometry & Calculus A
Math 242 Analytical Geometry & Calculus B
Math 243 Analytical Geometry & Calculus C
Math 341 Differential Equations with Linear Algebra I
Math 342 Differential Equations with Linear Algebra II
Maths subjects for Mechanical Engineering
Math 241 Analytical Geometry & Calculus A
Math 242 Analytical Geometry & Calculus B
Math 243 Analytical Geometry & Calculus C
Math 351 Engineering Maths I
Math 352 Engineering Maths II
Math 353 Engineering Maths III

Math 241: Analytical Geometry & Calculus A

Functions, limits, continuity, derivatives and definite integrals. Exponential and log functions; simple differential equations modeling exponential growth and decay (linear and separable ODEs).

Credit Hours: 4

Requires two years of high school algebra, one year of geometry and trigonometry.

Math 242: Analytical Geometry & Calculus B

Exponential and log functions; inverse trig functions; integration techniques; parametric curves; polar coordinates; infinite series. Includes use of the computer package, Maple, to perform symbolic, numerical and graphical analysis.

Credit hours: 4

Pre-requisites: Math 241

Math 243: Analytical Geometry & Calculus C

Vectors, operations on vectors, velocity and acceleration, partial derivatives, directional derivatives, optimization of functions of two or more variables, integration over two and three dimensional regions, line integrals, Green's Theorem. Includes use of the computer package, Maple, to perform symbolic, numerical and graphical analysis.

Credit hours: 4

Pre-requisites: Math 242

Math 341: Differential Equations with Linear Algebra I

Topics include first and second order differential equations, systems of algebraic equations, determinants, vector spaces, eigenvalues and eigenvectors of matrices and systems of differential equations. Emphasis on the interaction between these topics and appropriate physical systems.

Credit hours: 3

Pre-requisites: Math 242

Math 342: Differential Equations with Linear Algebra II

A continuation of MATH341. Topics include series solutions, Laplace transform methods, boundary value problems, orthogonality, higher order equations, difference equations and numerical techniques. Continued emphasis on the interaction between these topics and physical systems.

Credit hours: 3

Pre-requisites: Math 341

Math 351: Engineering Mathematics I

Solutions of linear algebraic equations, Gauss elimination, vector spaces, subspaces, linear dependence, linear ordinary differential equations of 2nd order and higher, initial value and boundary value problems, eigenvalues, coupled linear ordinary differential equations, nonlinear differential equations, with engineering applications.

Credit hours: 3

Co-requisites: Math 243

Math 352: Engineering Mathematics II

Laplace transform, application to constant coefficient ordinary differential equations, scalar and vector fields, Laplacian, line integrals, divergence theorem, Stokes' theorem, Fourier series, orthogonality, diffusion equation, Laplace's equation, wave equation, separation of variables, with engineering applications.

Credit hours: 3

Pre-requisites: Math 351

Math 353: Engineering Mathematics III

Numerical Methods in engineering, linear and non-linear algebraic equations, numerical solution of ordinary differential equations, Runge-Kutta methods, boundary value problems, finite differences, diffusion, Laplace equation, applications to engineering problems with programming.

Credit hours: 3

Pre-requisites: Math 351

Appendix 2: Literature Review

Mathematics Education for 21st Century Engineering Students

Literature Review

June 2007
Annabelle Lopez

Editors: Simi Henderson &
Graham Keen



Australian Mathematical Sciences Institute

This literature review is part of a project sponsored by the Carrick Institute for Learning and Teaching in Higher Education



Contents

Executive Summary	1
1. Introduction	2
2. Why is Mathematics Education Important for Engineers?	4
2.1 The Need for Improved Performance in Engineering Mathematics Education	4
3. What are the Most Prominent Teaching Methods Currently in Use?	6
3.1 Active Learning	6
3.2 Advanced Computer Based Methods – Use of Web Based Interactive, Software Applications and Programs	6
3.21 Which Software program?	7
3.22 Does Computer Based Learning have any Evidence of Success?	9
3.3 Addressing Student Variability	10
3.4 Multidisciplinary Approach	12
3.5 Problem Based Learning (PBL)	14
3.51 Does Problem Based Learning have any Evidence of Success?	14
3.6 Variations and Combination of Methods	16
3.61 Design-Based Learning	16
3.62 Projects-Based Learning	16
3.63 The “Integrated” Approach	16
3.64 The Interdisciplinary, Multi-semester Integration	17
3.65 The Four-Leaf Clover Model with Emphasis on Mathematical Modelling	19
3.7 Structuring by Learning Outcomes	20
4. How Are the Findings Summarised?	21
4.1 Impression of the Body of Literature	21
4.2 Discussion of Key Ideas	22
5. Conclusions	26
References	28
Appendices	
Appendix A	ii
Appendix B	iii
Appendix C	xi

Mathematics for 21st Century Engineering Students: Literature Review**Executive Summary**

Four groups of strategies for teaching engineering students appear in the literature. These are problem-based learning (PBL), multidisciplinary approaches, computer-based methods and strategies that address student variability. Each of these addresses one or more of the issues facing 21st century mathematics educators. Specific issues addressed are: student variability, the need to enhance learning, the need to keep abreast of technological advancement and the need to acquire 'soft skills' and to relate learning experiences to workplace situations. Reports that include results of formal quantitative studies are limited. The effectiveness of computer-based methods and PBL have both been quantitatively analysed relative to learning enhancement. The integrated method's success has also been supported by formal studies. It is useful to combine or integrate computer-based approaches and PBL, with or without physical models.

1. Introduction

Like most other professions, engineering has evolved significantly in the last two decades. It is continually changing to reflect the needs of the 21st century. The needs of engineering students change similarly. Not only have students' needs changed but their needs are more varied as well.

The engineering profession is re-examining its relationship with its scientific roots. For the mathematics discipline, which impacts not only science but also business and management, this is an exciting time to invigorate the curriculum and its educational approaches. This will first require a review of current thinking by engineering and mathematics educators around the world. A picture of current trends emerges not only from published books and journals but also from websites of recognised centres of education. Therefore this review refers to formally published articles, reports from professional associations and to accounts of educational practices that are well documented on websites.

The curriculum and educational practices must be responsive to rapid economic and technological developments as well as emerging global environmental and social problems. Students' personal needs must be balanced against changing requirements of industry and professional accreditation bodies.

In the last five to ten years alone, some of these changes have affected Australian universities in general and engineering curricula in particular. Continuous increase in mineral and oil price resulted in further growth in an already healthy resources sector. This was coupled with a buoyant and fast growing Chinese economy and recovery in the Japanese market (both countries are heavy importers of LNG, for instance) that resulted in higher demand for petroleum, mining and process engineering graduates.

Enrolment data reflect a more diversified student population with varied needs. Consequently, university student support arrangements that were not essential before are now considered important. Examples of these are additional student financial aid and provision of childcare facilities. More students have full time or part time work commitments partly due to higher university fees. Although the Higher Education Contribution Scheme (HECS) was first introduced in 1989 [1], its impact has been seen more during the last decade, in combination with other socio-economic factors. It is increasingly common for students to find employment after finishing high school, work for a few years, and then resume studies entering universities at more mature age.

In response to industry specifications, there has recently been a requirement for emerging professional topics to be included in the engineering curriculum. Examples of these are project management, human resource management and teamwork, communication skills, report writing skills, ethics, time management and environmental protection. These competencies are sometimes referred to as “soft skills”. [2]

It is therefore of paramount importance for educators of engineers in the 21st century to recognise changes in the engineering profession as well as changes in engineering students and to modify subjects to suitably cater for their new requirements.

Mathematics for 21st Century Engineering Students: Literature Review

The purpose of this literature review is to bring to light some of the more prominent teaching methods and strategies currently used to teach mathematics to engineering students in various universities worldwide. Emphasis is given to the effectiveness of these methods and strategies. Likewise of importance are extended learning environments such as drop-in centres and other associated learning support. The effectiveness of these methods, strategies and learning environments are discussed. Detailed discussion is found in Section 2 and a summary of key ideas is included in Sub-section 3.2.

A review of research on the teaching and learning of mathematics in Australian and New Zealand universities (from 2000-2003) inclusive found that most articles “contain little reflection, little evaluation or awareness of previous work in the area” [89]. There was much debate in the research about the use and extent of use of computer algebra (CA) systems in the mathematics community, however most statistics courses already rely heavily on computer software, with little work done by hand. The changing nature of students and the effect of this on the transition from high school to tertiary mathematics study is also widely discussed. There is also some research on the way students understand particular areas in mathematics. Wood [89] found gaps in the research, finding few articles researching the teachers, looking at higher level tertiary mathematics, cross-disciplinary teaching and learning, studies of graduates of mathematics and those who use mathematics, emotion and motivation in university teaching, alternative teaching and assessment methods.

This literature review is part of the initial stages of a scoping investigation to review teaching methods and learning environments employed in the teaching of mathematics to engineering students in Australian universities. Funded by the Carrick Institute for Learning and Teaching in Higher Education, this initial scoping project is headed by Philip Broadbridge of the University of Melbourne.

2. Why is Mathematics Education Important for Engineers?

Mathematics education is of such importance to the engineering curriculum because it helps to lay the foundation for good analytical and problem-solving skills often required in traditional engineering work. Mathematics subjects are prerequisite to a number of engineering subjects.

According to existing literature, mathematics skills are indispensable to engineering graduates. Furthermore, it is claimed that these mathematics skills are in fact not strong in many professional engineers and should be strengthened. This is hoped to be achieved through improved teaching methods. The need to reinforce these mathematics skills is further highlighted by a recognised engineering skills shortage not only in Australia but in many western countries. Methods used to enhance learning should also respond to issues such as diversified student population as well as technological and other changes that impact engineering mathematics education in the 21st century.

The importance of solid mathematical education for engineers is stressed by a number of authors [3-4] and argued by Blockley and Woodman [5]. It has been highlighted that mathematics is needed in engineering because “all mathematics is the ultimate form of logical rigour. It is the language of scientific communication, hence without a facility in mathematics engineers are cut off from scientific change and development.” Most civil engineers believe that only few mathematical concepts are used at work, Kent and Noss propose an explanation [6-7] as to why this is so. It has been suggested that this is because mathematics has become wrapped up in engineering practice. As an example, geometry and trigonometry have become so much embedded in engineering practice such that a structural engineer tends to think about say, a simple plane curve, in terms of what they mean in structural terms rather than in mathematical terms.

Furthermore, Blockley and Woodman [5] claim that with the advent of computers, emphasis has changed from the ability to perform engineering mathematical calculations to the ability to interpret the meaning of mathematics in engineering. This is especially true in the use of computer software applications. Blockley and Woodman’s idea is supported by Hadgraft [8] and Kent and Noss [9-10]. It has been established by Kent and Noss [9] that manipulative skill is less regarded than the “holistic” understanding or interpretation of mathematics and in identifying which areas need mathematical applications in an engineering context.

2.1 The Need for Improved Performance in Engineering Mathematics Education

Engineering students are seen to lack the necessary mathematics skills when they enter traditional technical occupations. According to Henderson [11], surveys of widespread practices in software engineering show that many engineers do not have sufficient skills to use discrete mathematics and logic as tools in performing their work. In the case of Civil Engineering, Blockley and Woodman [5] found that while it is paramount for engineers to understand structural behaviour, practising civil engineers have been known to make incorrect assumptions in finite element modelling whereby non-existent boundary conditions and degrees of freedom have

Mathematics for 21st Century Engineering Students: Literature Review

been set. This clearly illustrates a lack of understanding in interpreting mathematics as a tool to model physical and engineering conditions – a substantial part of what a young engineer’s work may require. Pollock [12] and Springer *et al* [13] have also expressed concern over students’ mathematical preparedness even as they enter higher education. This further highlights the need to strengthen engineering mathematics education at the university level.

In reference to the federally funded *Engineering Tomorrow’s Engineers*, a national project funded by the Collaborative and Structural Reform Fund (CASR) designed to address the current skills shortage of engineers and build Australia’s capacity to produce engineers with the skills required for the future, federal Education, Science and Training Minister, Minister Bishop was quoted saying:

“Australia’s strong economy has led to increasing demands for skilled workers in certain industries. A number of the projects I have approved from the 2006 funding round will assist further in addressing these skills issues”. [14]

The national engineering skills shortage has recently been formally recognised by the government. Funds have been directed to address this shortage and as a result additional engineering university places have been approved. In July 2006, Hon. Julie Bishop announced the addition of 510 new engineering places in universities from 2007. Prime Minister John Howard declared later in October 2006 an additional 500 Commonwealth-supported engineering places at universities from 2008. These extra places [16] would be invested over four years and will cost the government \$56 million.

Tilli and Trevelyan [15] have highlighted the importance of engineering work due to its substantial contribution to any country’s economy, indicating the major significance of an engineering skills shortage.

Projects that deal with an engineering skills shortage have also been accepted. One such project approved by the federal government to address the issue of engineering skill shortage is Australian Technology Network’s (ATN) “Engineering Tomorrow’s Engineers”. [14]

Circumstances overseas do not appear to be very different. It has been reported [17] that the UK Technology Colleges Programme and the New Zealand Science and Technology Teacher Fellowship Scheme have both been developed with the purpose of advancing science and technology education, as well as responding to the impacts of globalisation in curriculum design. Attention to student enrolment in STEM (Science, Technology, Engineering and Mathematics) due to severe shortage of graduates in many Western countries is also identified by van Langen and Dekkers [18]. In a separate report, Kent and Noss [9] presented statistics of declining engineering enrolments in the UK from 1988 to 2000. Generally, industries in Europe have been said to be discontented due to the low number of engineering students passing mathematics exams. [19] Graduates of engineering in many developed countries are regarded to be low in number, with only 6.4% of graduates noted to be in the engineering field in the USA in 2001, while the UK produced 10.5% of its graduates for the engineering labour market. [18] Engineering accounts for 8% of total graduations in Australia. [20]

3. What are the Most Prominent Teaching Methods Currently in Use?

From a survey of recent (from 1995 up to the present time) literature in mathematics education for engineering students, the following are notable means for teaching and learning to adapt to 21st century needs and conditions:

- use advanced computer based methods – web based interactive, software applications, or both
- address student variability
- take a multidisciplinary approach
- use a Problem Based Learning (PBL) strategy

These methods and features are discussed in more detail in the succeeding sections. Variations and combinations of these teaching methods are also discussed. Where evidence of any method's success has been reported, this is included, together with the discussion of the method's features.

Active learning deserves special mention because it is related to many teaching methods including computer-based and problem-based learning.

In addition to the most prominent categories of teaching methods (listed above), formulating subject objectives with the use of learning outcomes may enhance learning by better defining objectives and specifying student activities.

3.1 Active Learning

Active learning broadly encompasses all learning driven by the learner [21-22]. It is “learning by doing”. It has been claimed that the more students participate in their learning, the more they achieve [23]. In general terms, active learning is differentiated from a traditional lecture whereby students are passive listeners. Problem based learning (PBL), “learning driven by problem” [24], is classified as active learning [25]. Computer based approaches, especially where interactive software is used by students, are also classified as active learning.

In their report, Sanz *et al* [26] discuss the development of a project at the University of Utah highlighting the benefits of an active learning environment. This project centres on a highly interactive multimedia module for science and engineering students where by numerous opportunities for “learning by doing” are highlighted with the use of graphics/animation, virtual labs and simulation software. Active learning is also advocated by other authors [23, 27-28] with various perspectives.

In the following sections, computer based learning and problem based learning will each be discussed. Due to the relationship of active learning to both of these methods, as each of them is discussed, active learning is also emphasized.

3.2 Advanced Computer Based Methods – Use of Web Based Interactive, Software Applications and Programs

With the rapid progress of computer technology in the last couple of decades, software applications and the web have become important elements of our daily

Mathematics for 21st Century Engineering Students: Literature Review

activities. This reality is also evident in engineering mathematics education. Computer technology and its applications have been incorporated in engineering mathematics subjects at most universities in Australia. Hence, while one or more other teaching methods are employed, it may be observed that they are often used in conjunction with web technology or software applications.

In this section however, the application of computer technology in teaching mathematics to engineering students is discussed by itself.

Many authors [7-9, 26, 28-29] have either asserted that computer based teaching should be supported or highlighted the strong presence of computer software applications and various forms of computer and web-based interactive learning environments in the delivery of mathematics subjects to engineering students. This complements the equally large proportion of engineers' professional work being reliant on computer software applications. Taraban *et al* [27] highlight the importance of examining computer-based learning because it represents a leading progression in contemporary teaching. On the other hand, there are reasons why "engineering mathematics courses have not undergone wide scale changes towards IT. These are the high cost of moving 'chalk and talk' mathematics teaching out of lecture rooms and into computer laboratories, and the lack of a common grounding in mathematical technology in school mathematics curricula" [9].

Electronic learning (e-learning) is often thought to be the new way to communicate mathematics. It is also believed to enhance comprehension and stimulate interest [19]. Particularly for engineering students and practising engineers, familiarity with a number of software packages is said to be vital for effective problem solving [30].

Some authors [19, 27] suggest that the use of interactive mathematics can provide more engaging learning materials. This will consequently attract more students to study mathematics (and engineering) and help stem the declining number of graduates [9, 17].

Aside from using packages, interactive mathematics may involve personalised, interactive documents on the web. It may also include the use of the web as an unconventional calculator using numeric, graphic and symbolic mathematics interactively [19]. As well as assisting learning, computers can also be engaged in assessment by way of "computer aided assessment" (CAA). CAA has been reported to increase students' confidence and to reduce their stress levels [12]. Furthermore, some authors [4] have suggested that it would be beneficial for educators to explore the potential of using technology (software tools) to improve the interrelation between mathematicians and engineers and for bridging their individual knowledge in a fresh manner.

3.21 Which Software program?

Brenner *et al* [30] assert that computer programs for engineering students must be simple to understand and easy to use. They should also be capable of achieving straightforward results. This will prevent the technical details of the computer program from taking away the focus of learning from the main subject matter. Waldvogel [31] has communicated along the same lines. He supports the appropriate use of modern mathematical software in teaching engineering but cautions that

Mathematics for 21st Century Engineering Students: Literature Review

software cannot take the place of basic understanding. He is also a proponent of MATLAB, which is believed to be a transparent and versatile in numerical work [33].

At the University of South Australia (UNISA) [32], online study mode is accomplished via the internet where interactive technological tools are available. Most mathematics subjects for engineering students introduce students to MATLAB [33].

Other notable users of MATLAB include the University of Canterbury in MATH/EMTH271 Mathematical Modelling and Computation 2 [59, 62-64] and the University of New South Wales at Australian Defence Force Academy (ADFA) in ECM1 Engineering Computational Methods 1 [60-61].

Curtin University [34-35] has incorporated the Maple programming language in its engineering program. It is intended that Maple will reinforce lecture topics for the students but they will also be made aware of Maple's limitations.

Kent and Noss [4] put forward Mathcad and MATLAB as useful tools for engineers for constructing models and for their own specialised applications. This is primarily because these tools allow accelerated prototyping. This means that, "initial modelling ideas can be investigated by the potential users, and the resulting interaction often leads to an improved match between the model and the users' requirements". Mathematica or Maple is thought to be less useful as they are more difficult to understand without thinking in "explicit mathematical terms" [4].

Proponents of Mathcad [36] refer to its ability to export and import data to external files. Users are also known to be able to build on previous projects by consolidating old and new models. Karady and Nigim [37] maintain that Mathcad is easily grasped by students. Part of the reason for this may be because formulas in Mathcad worksheets appear in the same form as they are seen in textbooks [36-37]. MATLAB has been chosen at the Jerusalem College of Technology firstly because it is widely used in industry, and secondly because of its "pronounced Linear Algebra orientation" [38]. MATLAB is believed to help make the theory more transparent to the engineering student by allowing the student to follow the theory's operation in a wider range of applications. Whereas mistakes in manual linear algebra exercises can slip away unnoticed, MATLAB also acts as an effective teaching tool by refusing to process incorrect statements. MATLAB is also said to be very useful when demonstrating graphs of functions and discussing limits [39].

Some universities [37, 39] have also chosen MATLAB or Mathcad due to their wide application in employment and industry.

Positive feedback has been obtained for a MATLAB guide [39] developed at the University of South Australia. The title of this 160-page guide is "A Focused Introduction to MATLAB". It aims to enhance the teaching and learning of mathematics and serves as a bridge between the software itself and the lecture material. Contents of the guide include an introduction to MATLAB features, worked examples and exercises. It is believed that the MATLAB worked problems are very effective. This is because they are problems that are taken from the textbook; therefore a student is able to compare the textbook solution to the MATLAB solution.

Mathematics for 21st Century Engineering Students: Literature Review

Keady *et al* [88] discuss the benefits of CAA systems, underpinned by Computer Algebra (CA) packages, used in the delivery of questions to students via the web in Australia. They argue that CAA systems become more common as departments have larger service classes. They focus on the benefits of *stack* (free and open source, underpinned by Maxima), *AiM* (free and open source except for the CA, underpinned by Maple), *mapleTA*, *WebLearn* (commercial, both underpinned by Maple) and *CalMaeth* (commercial, underpinned by Mathematica), but recognise that CAA systems will continually change. They are proponents of Maple as the underpinning CA and argue that sharing systems, question databases, etc is crucial for progress to be made and to allow the systems to be economically viable.

Brenner *et al* [30] and Morgenroth *et al* [28] envisaged the inclusion and effective use of modelling and simulation in the fields of environmental, chemical and biotechnology engineering through the application of POLYMATH. POLYMATH is a numerical computation package used for modelling and simulation, particularly created for engineering students and engineering professionals. It is generally used for interactive problem solving involving algebraic equation systems, differential equations and regressions. Its latest version allows automatic migration to Excel. With the application of a few elementary rules, POLYMATH models can be transformed to MATLAB functions.

POLYMATH is easy to understand and simple to use. “It requires only minimal user intervention in the technical details of the solution process.” [30] POLYMATH is used in Ben Gurion University and in many other universities worldwide, including University of Adelaide and Curtin University of Technology, in Australia.

3.22 Does Computer Based Learning have any Evidence of Success?

Most authors and proponents of the use of software applications in engineering maths subjects do not have any formal quantitative empirical evidence of its success. At best, Brenner *et al* [30] offer some reasoning behind their conclusion stating the positive effects of the use of POLYMATH. They also demonstrate its effectiveness by describing how it was applied to the modelling and simulation of a classic problem in “Water Pollution Control”. Similarly, Mtenga and Spainhour [36] provide sound arguments to support their claim about the benefits of using Mathcad. They also illustrate the functionality aspect of Mathcad by presenting a problem (“Stepped Column”) where it has been implemented. Naimark [38] uses his observation to compare topics with and without MATLAB support. He explains that topics incorporating MATLAB are grasped well, with nearly all of the exercises solved correctly by the students. Colgan [39] bases his conclusions on student grades. But contrary to Naimark’s results [38], student grades indicate that mathematical knowledge and skills gained in subjects with MATLAB are comparable to those gained in subjects without MATLAB. It is the ability to program in MATLAB that increased significantly. Lee and Lin [40] showed how MATLAB might be integrated into an electrical engineering subject for the purpose of visualising and appreciating its application in a real environment, and only declared the associated relevance of measurement and analysis based on it. Educators generally make statements about how computer aided teaching and software applications enhance learning in engineering education, but do not provide any back-up quantitative information to support these claims [26, 28].

Mathematics for 21st Century Engineering Students: Literature Review

Taraban *et al* [27] recently conducted a case study comparing the effectiveness of software implemented interactive (active) learning compared to computer based plain text (passive) approach. The case study had twenty-five participants in a thermodynamics subject. Materials used were computer-based instructional supplements whereby screens display text, tables and charts, or interactive exercises and problem solving, modelling, simulation and online quizzes. As part of a method known as “verbal protocol analysis”, students’ verbalisations were captured as they viewed the computer screens. Verbalisations were associated with thought processes. Results of their case study showed significant increase in cognitive activity in interactive screens compared to text-based screens.

Results from the Taraban case study [27] indicate that computer based learning is much more effective when it is interactive than when it is passive. However, it is not categorically shown in their study that computer based interactive learning is superior to the traditional lecture and textbook based method.

The integrated approach has been used in a number of American universities. [41] This approach encourages active learning and combines different models mostly consisting of the physical model, mathematical model and computer simulation model. Electronic equipment, devices and machines, including computers, are widely used especially in measuring, collecting, storing and analysing data. Computer applications are part and parcel of this approach.

Nirmalakhandan *et al* [23] present recent findings involving independent evaluations of the success of using these active learning environments in engineering classes. Evidence shows that the integrated approach, promoting active learning, helps to improve engineering education with 81% of students confirming the effectiveness of the computer model.

3.3 Addressing Student Variability

Student variability has a number of implications. The student population is diverse in many ways. Due to the multiple needs and commitments of students, additional and diversified support is required. Examples of this are; flexible scheduling, childcare, financial aid, and support for students with disability. Diversity also means differences in students’ learning style and implies a wide range of aptitude among student intakes.

It has been documented in Australia and elsewhere that student populations are becoming more diverse [42]. For instance, women’s enrolment in engineering has been reported to increase in many countries worldwide [43]. Houghton and Dunne [44] and other authors [42] suggest that in order to address the challenges associated with this diverse population, students need to assume an active role in their learning. This may be associated with the belief that educators should pay attention to different learning styles as this is crucial to the success of any teaching and learning method [45]. These ideas are related because when students actively participate, they have more opportunity to activate whatever learning style is suitable for them.

Townend [46] makes another proposal, specifically dealing with varying abilities of engineering students, to make the students’ mathematical encounters more “user friendly”. Croft and Ward [3] present a similar solution to take into account the individual needs of students with the use of an open environment and include

Mathematics for 21st Century Engineering Students: Literature Review

components with flexible study arrangements that are considered to enhance and stimulate learning.

One such open environment is the Maths Learning Centre [47] at Loughborough University, which provides, among other things, one-to-one teaching and 24 hour, 7-day on-line support. The presence of learning centres is supported by Blair-Editor [42] and Fuller [48]. It is asserted that students of mathematics need a supportive extended learning environment such as learning centres, tutoring labs, counsellors, support for students with disabilities and others.

Bamforth *et al* [49] describe a pre-session course run for electrical engineering students at the University of Loughborough with non-conventional maths backgrounds (without A-Level) to help and retain students at risk of underachievement. Aside from mathematics lessons, this course also covers team and key skills. The course is a good starting point because it helps at-risk students become aware of the facilities and support available to them at the university and it also helps them begin the process of improving their main skills. It has been noted that student feedback has been largely affirmative.

As a whole, the provision of varied learning resources and support such as those found at Loughborough University and Central Queensland University [48] gives more opportunity for a variable student population to learn in the way that best suits them.

In order to address the issue of students having multiple life demands such as study, work and family, the University of South Australia offers flexible study arrangements [50] to students to help them cope with the demands of studies and of other areas in their life. Students are able to take subjects on campus, externally, online or a combination of these. Online students (in a computer science subject) have been shown [51] to perform at least as well as students in a traditional lecture. In a separate report, Hadgraft [8] has specifically suggested that skill development in engineering subjects should be supported by online assessment in order to provide more flexibility for students to study and obtain assessment at their own pace.

The University of Western Australia (UWA) is claimed to have a diverse student population. Based on its website [52], its student population is said to be diverse in relation to cultural background, age, gender, race, disability, sexual orientation and socio-economic status. According to UWA's Best Practice Pathways Database [53] completed in 1998 and 2000, the Department of Mathematics and Statistics takes into consideration the diverse learning needs of students by using different forms of delivery such as; lectures, computer-based learning (including videoconferencing) and use of the web. Web-based materials can be accessed anywhere in the world at a time convenient for the students. Examples of materials placed in the web are subject notes, assignment solutions, software packages and assessments. The Department also uses various forms of assessment such as; written assignments, tests and quizzes, computer-based tests, reports and essays. Students are often required to use computer packages and the computing lab. The Mathematics Learning Centre is another option for students.

Based on information provided by Professor Mario Zadnik [54], Dean of Teaching and Learning at Curtin University, it can be seen that Curtin University has recognised the variability in knowledge and competencies of students entering the

Mathematics for 21st Century Engineering Students: Literature Review

engineering program. They have therefore introduced two Mathematics subject groupings or series, one intended for students who achieved better marks in their Year 12 studies (Engineering Maths 110 then 130 series with more challenging and enriching contents) and another series (Engineering Maths 120 then 140) for students whose marks were lower.

Similar streaming is being implemented at Edinburgh University [55], with one stream provided to students from average academic background and another stream functioning at remedial level. Different teaching approaches and different assessments are used to support the streaming being implemented at the University of Liverpool [56]. At ETH Zurich [31], streaming is being implemented with mathematics taught to first and second year engineering students. Students are however able to choose between a fast Stream A and a slow Stream B and this is certainly contributing to additional flexibility of their programs.

Although the success of University of Liverpool's streaming by ability is only gauged informally through staff-student forums, they have generally been judged to be very good [56].

3.4 Multidisciplinary Approach

The LTSN Maths Team [57] recommended that mathematics subjects to engineering students be delivered within an engineering context. One way to help students integrate their knowledge and be able to see mathematics in the context of engineering is to design a subject where collaboration between engineering and maths departments (and other staff) is a fundamental principle. Additionally, Haryott [58] suggests that for a good working engineering syllabus to succeed, collaboration should be accomplished not only among academics but it should also include accrediting bodies and recent engineering graduates.

Maths and engineering collaboration is undoubtedly consistent with the arrangement in modern industrial working practices [4]. It has been identified (and success noted from student comments) that interface between engineering and mathematics subjects can be achieved through the multidisciplinary approach. This was evidenced by the noted success (based on student feedback) of this program at the Edinburgh University [55] and at Canterbury University [59].

In Australia and in New Zealand, Dr. Steve Barry (ADFA) [60-61] and Prof. Graeme Wake (formerly from University of Canterbury) [59, 62-64] have independently helped introduce the multidisciplinary approach in teaching Numerical Methods. Groups usually working together include mathematics, engineering and either statistics or computer science academics.

In the Canterbury model [59, 62-64], each week generally contains lecture and lab sessions handled by the Department of Maths and Stats. Lab sessions are used for problem solving. However four out of the twelve weeks (four weeks are non-consecutive) are devoted to engineering case studies that are taught by staff with an engineering background. The case studies are carried out in groups of two students.

The ADFA model [60-61] is different in the way the interfaces are structured. The whole subject consists of six engineering problems based either on real world occurrences or on more advanced engineering subjects. It may be said that the subject

Mathematics for 21st Century Engineering Students: Literature Review

has strong elements of PBL, although individual students do assignments. Development and delivery of the subject is accomplished jointly. Different departments present each lecture and each lab session jointly. Positive feedback has been received from student survey results as well as from staff-student review meetings. Although fundamentally multidisciplinary due to the collaboration of different departments in the subject's delivery, the teaching strategy employed in this subject in Numerical Methods is a mix of lectures, computer based laboratory and "real world" engineering problems/case studies (PBL). There is substantial interface between mathematicians, engineers and computer scientists in ADFA's model [60-61] with each lecture and each lab session jointly handled by three educators. On the other hand, Canterbury's model [59, 62-64] has additional support for students in the form of dedicated tutors, regular drop-in help classes, consultation periods, availability of web-based assistance and help with administrative issues. Both subjects have used MATLAB and both have claimed success based on student feedback. Both subjects have also met a number of challenges related to implementation. Both subject designs have been newly implemented (2003) and no measurable analysis has been done yet on their effectiveness.

These approaches may be effective because materials are presented in different ways (lecture, lab and case studies) within a short time frame and this enables the students to obtain thorough and in-depth knowledge/skills of the subject matter. The presence of regular lab work (weekly) is useful because the students' lab experience will support and complement what has been learned in the lecture each week. This will also reinforce learning and provide hands-on opportunity with MATLAB and other computer packages.

There are various ways in which collaboration between maths teachers and engineering departments benefits students at the ETH Zurich [31]. In this university, engineering departments are consulted and their agreement sought when determining topics for second year engineering maths subjects. Practical examples given in class are also taken from engineering fields. Taking practical examples and problems from engineering has also been suggested for technological colleges in Japan [65], although this idea is taken further by proposing to revisit these problems at a later time. Waldvogel [31], in his presentation at the IDEA League 'Workshop on Mathematics in Engineering' proposed additional improvement by focusing on and extending linear algebra and numerical analysis with possible inclusion of discrete mathematics, due to the fact that practical engineering problems "are rarely solvable in closed form".

Since 2004, Zadnik [54] of Curtin University of Technology has chaired a committee composed of representatives from maths and engineering. The committee aims to tackle issues in mathematics teaching, to bridge the maths-engineering divide and to make proposals for improvement in the following areas:

- Incoming students' variable knowledge and competencies
- Engineering maths subject contents and learning outcomes
- Who teaches
- Guest lectures by engineering staff reinforcing the importance of maths

Incorporating guest lectures by engineering staff is an innovative teaching practice for mathematics subjects and this can potentially improve engineering students' appreciation of mathematics [54]. This is because the engineering staff can reinforce

Mathematics for 21st Century Engineering Students: Literature Review

the importance of maths and better relate mathematics to the engineering concepts, practical problems and even work requirements.

3.5 Problem Based Learning (PBL)

Active learning is “learning by doing”. This occurs when students play an active role in their learning. The problem based approach is a form of active learning where learning is driven by the problem. Some authors [42, 44] suggest that in order to address the challenges associated with a diverse student population, students need to assume an active role in their learning.

Although it has been identified from Kent [66] and Tilli and Trevelyan’s [15] research that engineering practice is not clearly understood, some authors [9, 27] believe that engineering has a creative quality. Furthermore, Haryott [58] advocates that in order to support the creative nature of engineering, the use of problem-based learning should be explored further for teaching mathematics to engineering students.

PBL has been claimed [67] to be the “natural technique” to use in teaching engineering because it duplicates work situations most engineers find themselves in. Workplace situations hold similarities with small group, PBL in the classroom. This is evidenced by the fact that in engineering work, people team up in small collaborative groups [13] and encounter problems that are open-ended and often with conflicting elements.

Litzinger *et al* [68] propose a revision of engineering curricula to include multiple learning experiences that challenge students to cultivate self-directed learning skills. PBL subjects have been studied in connection with this and positive correlation was obtained between PBL and readiness for self-directed learning. Aside from multiple learning experiences, some authors [67] have suggested PBL based programs include more student participation. It has been proposed [69] that there are many opportunities for engineering students to work on projects requiring their collaborative skills. This is demonstrated by the Faculty of Engineering at the University of South Australia [39] and ADFA [60-61]; group work and report writing are included in mathematics subjects for engineering students. These subjects are intended to be problem-based, with students encouraged to acquire additional knowledge on top of what was covered in lectures. They also require group work and report writing to assist in developing skills associated with working cooperatively, time management and investigating “real applied problems”. This is supported by the PBL experience described by Johnson [70] in a hydraulic engineering class.

Although benefits of PBL have been reported to be positive [71], Johnson [70] has pointed out that there is considerable time associated with the overall implementation of this strategy. Contrary to this, Jonassen [72] argues that online environments are able to provide a platform for designing, developing and implementing PBL with minimal support needed.

3.51 Does Problem Based Learning have any Evidence of Success?

Morgenroth *et al* [28] have suggested the usefulness of PBL in understanding and appreciating mathematical modelling. This claim is supported by Thomas [73] with the success (based on feedback) of a game show format for PBL classes in mathematical modelling. Projects undertaken by students have been argued to

Mathematics for 21st Century Engineering Students: Literature Review

enhance learning; however the claim is usually based only on direct students' feedback [70, 74-76].

Karady and Nigim [37] report on a proposed problem solving method in power engineering at Arizona State University which has been tested for four years, however no quantitative data is put forward. They claim that this method, which also utilises Mathcad, has helped improve grades and has also lifted student interest in the subject matter. It is postulated that the success of this problem solving method can be attributed to the interactive nature of the process. This signifies that it is effective for students to discover the trends, meanings and interconnections in an interactive way.

An integration of studies from 1980 to 1999 was accomplished by Springer *et al* [13]. This pertains to small group learning's effectiveness with undergraduate students of science, mathematics, engineering or technology (SMET) in North American educational institutions. This integration study analysed 39 reports using the standardised mean difference (d-index) effect size. The d-index was measured for the three most prominent learning outcomes found in SMET education literature. These are: academic achievement, persistence (or retention) and attitude.

As guidance for the interpretation of the d-index, a "d" value of 0.20 is considered to have a small effect, $d=0.50$ can be taken to have moderate effect, and if the value of "d" is 0.80 then this means that the effect is large. An effect size of 0.33 is considered to be the minimum required to confirm significance, and typical values range from 0.25 to 0.50 [13].

"The main effect of small group learning on achievement, persistence, and attitudes among undergraduates in SMET was significant and positive" [13]. Results of the integration study are detailed below:

- Achievement "d" value of 0.51, more than the average value 0.40, would move a student from 50th percentile to 70th percentile;
- Persistence "d" value of 0.46 would reduce attrition in SMET subjects by 22%;
- Attitude "d" value of 0.55 exceeds the average value of 0.28 for classroom-based educational interventions on affective outcome measures.

The significance of this analysis to PBL cannot be ignored if small group PBL is utilised.

Taraban *et al* [27] performed a case study on 25 science and engineering students of thermodynamics which yielded positive results for effective use of software implemented active learning. The case study examined cognition when students were placed in a rich visual, auditory and print environment. Students' verbalisations are sampled while they interacted with and solved problems using software. While this experiment has e-learning elements associated with it, it is actually the interactive aspect of it that is being examined. The method used is considered to be PBL because problem solving was accomplished while learning. The kinds of learning materials used support theories of skill progression, which require that students have the knowledge first and then the technique to translate that knowledge into skill by applying it to problems. The materials are different from a traditional lecture in their ability to preoccupy senses and give immediate feedback to student input. The more interactive the material is, the more student participation is required.

Mathematics for 21st Century Engineering Students: Literature Review

It is noted that while interactive and PBL are both forms of active learning, they are not exactly the same as each other but only contain common elements. As shown in the experiment mentioned in the previous paragraph, they can be designed to blend together.

Innovative teaching methods being used in a number of American universities [41] are consistent with an integrated approach characterised by active learning environments. This approach encourages “learning by doing”. Projects are often required of students working in small groups, challenging them to “rationalise, reconcile, predict and validate” theoretical knowledge against the physical model [23]. Active learning can take several forms and one of these is PBL. This integrated approach can therefore be thought of as an extension of PBL.

Nirmalakhandan *et al* [23] present recent findings involving independent evaluations of the success of using these PBL based learning extension in engineering classes. Evidence shows that this integrated method helps to improve engineering education, with 92.1% of respondents attesting to the effectiveness of this teaching strategy.

3.6 Variations and Combination of Methods

3.61 Design-Based Learning

The general concept used in Design Based Learning (DBL) is similar to that of Problem Based Learning (PBL). In PBL, the problem drives learning, that is, the problem is posed first before learning can begin. Similarly in DBL, the need for learning is met only when it is necessitated by design. The reasoning behind both approaches is the same: Students are more motivated to learn if they know ‘Why’ they are doing it.

Kent and Noss [9] suggest that mechanisms for change are ripe, they promote a so called “pull-based mathematics” and “design-based approach to engineering subjects”:

“Perhaps common ground can be gained by a constructive dialogue on two fronts: on the mathematical topics in the curriculum, and on delivery and pedagogical approaches. These two issues are intertwined, and consideration of one without the other leads inevitably to misunderstanding and inertia. Engineering subjects are tending towards ‘design-based’ approaches (employing design as an organising principle), with a decreasing use of ‘chalk and talk’ pedagogy, and mathematics subjects will have to accommodate themselves to this trend (this does not necessarily entail a radical departure from existing mathematics teaching). Although a number of methodologies are being chosen for design-based learning, the effect on mathematics is similar: the need for analysis is ‘pull, not push’ – the need can emerge where design requires it, not pushed into the student prior to having a meaningful context for it” [9].

3.62 Projects-Based Learning

In 2003 “Projects Based Learning in Engineering” (PBLE) published a guide [77] that advocates PBLE. Due to their similar descriptions and acronym, PBLE and PBL can be easily confused with each other. Educators’ implement PBLE with the “use of projects in their work with students”, and not just any type of problem. Projects may

Mathematics for 21st Century Engineering Students: Literature Review

include environmental impact assessment, design portfolio and simulated public enquiry. In a separate report [71], it has been suggested that projects based learning will be more straightforward for academics to adopt due to their familiarity with projects.

3.63 The “Integrated” Approach

The integrated approach has been found to be used in a number of American universities [41]. This approach encourages active learning and combines different models mostly consisting of physical, mathematical and computer simulation models. Electronic equipment, devices and machines, particularly computers, are widely used especially in measuring, collecting, storing and analysing data. Projects are often required of students working in small groups, challenging them to “rationalise, reconcile, predict and validate” theoretical knowledge against the physical model [23].

Nirmalakhandan *et al* [23] present recent findings involving independent evaluations of the success of using active learning environments in engineering classes. Hydraulic engineering classes totalling 131 students over 5 semesters were taken for the evaluation. The effects of an integrated approach promoting active learning using a combination of physical, mathematical, and computer simulation models were measured using student surveys and student performance. Collated end-of-semester survey results from five semesters indicated that 90.6% of students attested to the effectiveness of the physical model, 80.6% of students confirmed that of the computer models and 92.1% affirmed that the teaching approach helped improve their problem solving skills. The percentage of students that received A, B or C also increased to about $86 \pm 4.7\%$ after using this approach for 5 semesters. This is an acceptable value compared to the percentage before using this strategy, which was $70 \pm 1.3\%$. Although this evidence may be thought to be limited, they show that the integrated approach promoting active learning helps to improve engineering education.

In a separate report, Duran *et al* [78] show that a learner-centred approach highlighting student satisfaction has a positive effect on learning. The Software-based methodology (SBM) has been designed, implemented and formally tested on students of Electrical Machines and Installations at the University of Seville. SBM has distinct similarities with the method tested by Nirmalakhandan *et al* [23]. Both methods are fundamentally integrations of physical, theoretical and computer-based models. The software simulations in SBM provide visual tools that help students understand theoretical concepts. Real examples in the laboratory complement the virtual scenarios. SBM is highly interactive not only with respect to the computer simulation but also in terms of promoting discussion and brainstorming. The effects on approximately eighty-five students (different pre- and post test) were formally tested using questionnaires and cognitive tests. Statistical evaluation yields positive results for this integrated method.

3.64 The Interdisciplinary, Multi-semester Integration

While strategies such as the integrated approach discussed in the previous section, the ADFA model [60-61] and the Canterbury model [59, 62-64] achieve the link “laterally” across theory, computer applications and physical models in one subject, Avitabile [75-76, 79] and his group’s approach at the University of Massachusetts Lowell integrates materials both “laterally” across modules/projects, and “vertically”

Mathematics for 21st Century Engineering Students: Literature Review

from earlier subjects to later subjects across several semesters. The simultaneous lateral and vertical integrations result in interweaving of learning materials. This interdisciplinary, multi-semester approach concludes with a final year project in a Dynamic Systems subject. The strategy uses modules that have been developed for inclusion in a number of lower year pre-requisite subjects.

The strategy can be thought of as having a large project spanning several semesters. The preceding phases of this project are the pre-requisite subjects from 2nd year to 3rd year and its final phase is the Dynamic Systems subject. All skills and knowledge obtained in the preceding phases culminate and are utilised in dynamic systems.

This innovative engineering teaching strategy has various modules integrating basic materials from pre-requisite subjects and problem solving materials in later subjects [75-76, 79]. Modules have been set up in pre-requisite subjects such as:

- Differential Equations (2nd Year)
- Mechanical Laboratory (3rd Year)
- Numerical Methods (3rd Year)

with topics including:

- Numerical Integration/Differentiation
- Visualisation Tools for Understanding 1st and 2nd Order System Response Characteristics
- Understanding Complex Frequency Response Characteristics
- Development of a Virtual Measurement System.

In order to make the connection between the interrelated subjects, modules have been developed and deployed to the preceding subjects [75-76, 79]. The injection of these modules enables the students to see the connection between earlier subjects and later subjects. The strategy is thus fundamentally a “vertical” integration of earlier materials and later materials achieved through the use of modules.

It should be stressed that as integration is achieved between lower year subjects and higher year subjects (“vertical”), to some extent different materials/modules are also being combined in each year (“lateral”). An example of this is the combination of virtual measurement system (PBL, computer-based), actual measurement system (PBL, computer-based, physical model) and supporting tutorial material [76]. Furthermore, the same mass-spring-dashpot system (physical model) used for actual measurement is also used in teaching second-order linear differential equations.

There are in fact two examples of projects that are being studied from different perspectives for four successive semesters. These are the simple RC circuit and the mass-spring-dashpot system. All of these result in an interweaving effect on students’ learning and appreciation of the relationships between materials from 2nd year, 3rd year and through to final year engineering [79].

It is also worth noting that the overall strategy of interweaving materials (“vertical”/“lateral” integration) relies on the use of PBL and computer applications for its actual implementation. Additionally it also incorporates physical models in the subjects and has extensive use of electronic equipment and devices, GUI (graphic user interface), as well as hands-on student activities [75-76, 79].

Mathematics for 21st Century Engineering Students: Literature Review

One such hands-on module is the “virtual measurement system”, which can be considered as preparation for the actual experimental project. Another such module focusing on real-world (real data) measurement is R.U.B.E. (Response Under Basic Excitation). This module is an actual measurement system having variable mechanical parameters and is available as an online experiment. “In this module, students collect and process data to numerically integrate and differentiate displacement and acceleration measurement. The measurement system forces the students to address issues related to real-world measurements” [76]. The actual system has variable mechanical parameters in that it is able to change data for every operation resulting in all data sets being different from each other.

Assessment of the first few semesters when this hands-on project was available shows that students developed more knowledge and understanding of the subject matter. This is evidenced by students’ direct comments presented by Avitabile [76].

Integration has been achieved in the Differential Equations subject [79] by comparing an analytically obtained solution to one attained by the use of modelling. The computer-based tools allow students to study particular systems that have previously been defined by theory. Both of these are validated against the physical model including actual measured data. The differential equations module is in fact being investigated again from various other viewpoints in three more semesters.

Theory from earlier subjects can be compared with application in later subjects [79]. Lecturers likewise remind students of materials studied in earlier subjects and discuss how these materials relate to current topics. This is achieved by comparing topics and modules and by highlighting particular techniques learned in earlier subjects that are crucial in later subjects. Early years’ subjects include projects in integration, differentiation and regression analysis (with MATLAB-based graphical user interface tool) that are vital for application in later years [75]. In the subject itself (for example, Regression Analysis), the traditional analytical method (hand calculations) can be compared to software-based methods of using MATLAB and Excel. Focus on distortions present in real-world data is achieved with the use of graphical user interface (GUI). GUI is useful for graphical presentation of data that can be manipulated by the students.

All of these material overlaps occurring in various subjects throughout the engineering degree program help to effectively integrate learning.

3.65 The Four-Leaf Clover Model with Emphasis on Mathematical Modelling

Ernest [80] has warned to exercise caution before accepting the effectiveness of the integrated or modelling approach. Mathematical modelling has been identified by Hadgraft [8] as a necessary constituent of engineering education particularly when applied to real world problem solving (combination of PBL and computer-based approach). It has been stressed further that reinforcing mathematical modelling skills can be effectively achieved by practising this technique many times. “Students need to be able to move from the modelling task to skill development and back again”. The skill development learning model is extended further to a four-leaf clover education model, where the additional elements include “learning from others” and “learning from literature”. Designed in consideration of situations usually encountered in

Mathematics for 21st Century Engineering Students: Literature Review

industry, the four leaf clover is a comprehensive representation of what is required in engineering education namely: modelling skills, mathematical technique skills, collaborative learning skills and research skills.

The four-leaf clover engineering education model [8] highlights modelling in a PBL-computer based combination method that provides learning experiences said to be close to workplace situations. The incorporation of various elements being addressed in this education model is a strong feature, as well as the emphasis on repetition and practice.

3.7 Structuring by Learning Outcomes

One way in which mathematics can be related to engineering subjects and engineering applications is by incorporating this statement in the learning outcomes [34-35, 81-82]. A direct and simple expression of a goal (Outcome) will necessitate certain activities to be designed in order to achieve the desired outcome. Examples of such learning outcome statements can be found in Curtin University's Unit Outlines for Engineering Mathematics 140, 120, 130 and 110 [83]. Below are examples of actual statements:

- The ability to perform routine vector and matrix manipulations which arise in engineering
- The ability to sketch and visualise elementary mathematical functions routinely used in engineering analysis
- Understanding the concept and role of functions in an engineering context
- Identifying the role of mathematics in your own and related discipline area.

Some examples of how engineering applications have been covered are:

- Learning Outcome of adding, subtracting and taking scalar multiples of vectors are being applied to situations such as determining angles between different members of a truss
- Learning Outcome for Applications of Integration includes determining the work done in compressing/stretching springs and in lifting objects as well as determining moments and centres of mass/centroids [34-35].

Subject guides [84-86] for mathematics at UWA indicate that to some extent, Outcomes-Based Learning is being utilised in those subjects. Lectures, group work, lab, practice classes and the use of computer applications are the main features and learning environments used.

Based on publications and other materials covered in this review, there have been no studies made showing that specification of outcomes leads to better student learning than more traditional curriculum description.

4. How Are the Findings Summarised?

4.1 Impression of the Body of Literature

Although articles collected for the literature review of this research project are mainly accredited published papers, also included are publications by legitimate professional organisations, subject materials from universities, presentations to scholarly workshops, official communication between academics and materials from academically recognised websites. Published papers are predominantly from 1995 to 2007.

A large number of published papers are about computer-based/technologically up-to-date methods. However it is useful to note that in many of these articles, the use of technology is combined with one or more other methods. It is common to find computer based methods intrinsically mixed with active learning (specifically PBL – usually small group).

There are also many papers written about PBL. Springer *et al*'s [13] paper on small group learning has been considered to be related to the PBL approach. This is because small collaborative groups characterise a common form of PBL.

The variability within the student population has been recognised by a number of authors. This has again been linked to active learning (or specifically PBL – usually small group). The multidisciplinary approach is the least written about, however this method has been gaining ground in Australian and New Zealand universities.

Most of the articles surveyed present a specific teaching method or discuss aspects of a particular method. Approximately half of these works include some discussion about the method's success (or failure) usually as related to learning enhancement, however this information is often obtained informally through student feedback or through the authors' experience and observation. Four papers provided some formal quantitative measure of performance or effectiveness of a method or a mix of methods. All four papers have been peer-reviewed. Three of these were published independently in January 2007. The first paper written by authors from University of Seville [78] presented results of a software-based teaching methodology but has strong elements of active learning. The second paper, written mostly by staff of Texas Tech University [27] examined the effect of an interactive screen for software implemented active learning. The third paper is from New Mexico State University educators and research staff [23], although their experiment was structured fundamentally around the integrated approach (physical, mathematical and computer models combined), the experimental results can also be established for effectiveness of computer based learning and active or PBL independently. The effect of small-group learning has also been determined quantitatively, and a meta-analysis performed with reports from 1980 to 1999. This was summarised in a highly cited paper (47 citations from 1999) written by Springer *et al* [13].

From the literature review, there are no reports that compare the performance of the different methods of teaching & learning in a measurable and controlled manner. Admittedly, this would be difficult to achieve. Like Wood [89], we also found limited research on the teachers of mathematics to engineering students, papers on the teaching of higher level mathematics - most research focuses on first and second year

Mathematics for 21st Century Engineering Students: Literature Review

service teaching and research on alternative teaching and assessment methods in the teaching of mathematics to engineers in Australian universities.

Included in the literature reviewed are a small number of highly cited and well-cited papers. One highly cited paper on small group learning has already been mentioned. Also referred to are six papers that are on average cited once every year (5 citations from 2002) and four other papers that have been cited once every other year on average (2 citations from 2003). The number of highly cited papers in this field is surprisingly low.

The work of Springer *et al* [13] has been cited by numerous writers and researchers due to the relevance and extensiveness of its analysis. This paper provides very useful material because it involves quantifiable information synthesized in a systematic way. In addition, it is relevant because it serves as a summary integrating studies from two decades.

4.2 Discussion of Key Ideas

This review of literature indicates that two categories of teaching methods are most frequently claimed to be effective. These are Advanced Computer Based Method and use of PBL.

Computer Method's claimed advantages are that it stimulates interest and it enhances comprehension. While many authors [26, 28, 40] argue this to be the case, most do not offer quantitative evidence to support their claim [30, 36, 38, 39]. At best, reasoning and conclusions are obtained by means of observation, experience and informal student feedback. Although few papers with empirical evidence have been found [23, 78], they show vital information for engineering maths teaching research. Computer based method also supports students' introduction to the profession, as most industries are reliant on computer applications [28].

An important aspect of computer-based method is that it is often used in conjunction with other methods. As open-ended engineering problems can be given to students and while the computer application being adopted is based on an interactive platform, computer method blends with the problem based learning approach.

Nirmalakhandan *et al* [23] provide interesting insight with the results of their study of 131 students of engineering over 5 semesters. Their study has measured the effectiveness (in terms of learning enhancement) of physical and computer models independently, and they have also quantified the effectiveness of integrating these models with the theoretical (traditional) model. However, the results of this study may have some shortcomings due to limited statistical analysis.

The experiment presented by Taraban *et al* [27] is about the comparison between interactive (active) and passive learning with respect to learning enhancement. Both of these learning environments are on online platforms. Although they have used established techniques utilising cognition and learning research as well as statistical analysis, data has been obtained indirectly and this may affect the validity of their final results. They have also used a smaller sample of 25 students.

Another experiment by Duran *et al* [78] shows the effectiveness of computer based approach in aPBL. The authors claim that increased student satisfaction leads to the

Mathematics for 21st Century Engineering Students: Literature Review

success of its implementation. Termed Software-Based Methodology (SBM), their method utilises virtual scenarios for theoretical explanations, all in a highly interactive environment where real examples and simulations are shown in lectures. Questionnaires and cognitive tests have been statistically analysed proving the effectiveness of this method in terms of learning enhancement.

Three computer software applications have been noted to be effectively used in universities as well as in industry. These are MATLAB, Mathcad and POLYMATH. MATLAB is perceived to be the most useful [4, 38, 39]. Mathcad is also well regarded [36, 37] and POLYMATH has started to gain a lot of support [30].

PBL is claimed to be a method that effectively addresses diversity [42, 44]. It has also been identified to support the creative nature of engineering [58]. Like computer-based methods, PBL is a good way of assisting students' initiation into the workforce because PBL environment is close to what engineers might encounter in their profession [67]. This is to do with the presence of challenging problems usually with disorganised and conflicting requirements [13]. If small-group learning is utilised in PBL, this is also similar to what engineers might experience at work, that is, being part of a small collaborative team.

Challenges associated with teaching a diverse student population can be assisted by using methods characterised by active learning [42, 44]. Active learning is also advocated by other authors [23, 27, 28] from various perspectives. Students are thought to achieve more when they actively participate. Studies that measured the effectiveness of active learning environments support this claim [23, 27]. PBL is mostly defined by active learning and there is therefore an additional benefit of this method. Small group PBL may be promoted based on a meta-analysis performed by Springer *et al* [13] on small group learning's effectiveness (with respect to learning enhancement) that yielded positive results. In addition, computer based methods are more effective when based on an interactive platform.

Integrated approaches, combining various methods, particularly physical, computer and theoretical (traditional) models are being employed in the US [41]. Nirmalakhandan *et al* [23] show quantitatively that the integrated approach can be effective in enhancement of learning.

The main advantage of integrating methods is that it allows educators to mix methods and capture the benefits of each one. Computer-based method easily blends with small-group PBL (or any other method) and there are some areas where multidisciplinary approach may be injected as well. Universities that use integration in the US also include the physical model. The presence of physical models provides real benefit, as shown by results from Nirmalakhandan *et al* [23].

The inclusion of physical models in a projects based small-group PBL utilising computers and electronics applications is providing unmistakable advantage to learning approaches represented in many American universities.

Another advantage of integration is the flexibility it provides. As students “see” a particular problem in different perspectives (via different models), they are given more than one opportunity to grasp the subject matter [75-76, 79]. Furthermore, this approach is able to address diversity, with students' differing learning styles catered for by various models. The third benefit of integration is its unifying effect of being

Mathematics for 21st Century Engineering Students: Literature Review

able to relate theoretical concepts to physical and computer simulation models. This will assist students see mathematics in the context of engineering and to appreciate the practical applications of theory.

While “lateral” integration of learning materials may be achieved across theory, computer applications and physical models in one subject, as evidenced by the ADFA model [60-61] and Canterbury model [59, 62-64], Avitabile *et al* [75-76, 79] have also shown that simultaneous “lateral” and “vertical” integration is effective. Their approach at the University of Massachusetts Lowell integrates materials both “laterally” across modules/projects, and “vertically” from earlier subjects to later subjects across several semesters. This strategy when applied to an assortment of modules and topics results in interweaving of learning experiences. This interdisciplinary, multi-semester approach concludes with a final year project in a Dynamic Systems subject.

In order to make the connection between the interrelated subjects and topics, modules have been developed and deployed in the preceding subjects. The injection of these modules enables students to see the connection between earlier materials and later subjects. The strategy is thus fundamentally a “vertical” integration of earlier materials and later materials achieved through the use of modules. However it should be stressed that to some degree, materials and modules are also being combined in each year. As subject elements get interwoven, so do the teaching methods/models employed. These often involve the use of PBL and computer applications. Some physical models are also utilised. Applications of GUI and electronic equipment and devices to projects and hands-on student activities are extensive. Documented direct students’ feedback has been positive [75-76, 79].

In Australia and in New Zealand, Barry (ADFA) [60-61] and Wake (formerly from University of Canterbury) [59, 62-64] have independently helped introduce the multidisciplinary approach in teaching Numerical Methods. In addition to collaboration of different departments, the teaching strategy contains a mix of lectures, computer based laboratory and “real world” engineering problems/case studies (PBL). Problem solving is performed through MATLAB. There is substantial departmental cooperation in the ADFA model with three or more academics teaming up to deliver each lecture and each lab session. On the other hand, the Canterbury model has additional support for students in the form of dedicated tutors, regular drop-in help classes, consultation periods, availability of web-based assistance and help with administrative issues.

Above-mentioned multidisciplinary-based subjects share a number of attributes. Both subjects encountered administrative and other challenges, however based on student feedback they have both been very successful in enhancing student learning [59-64]. The two subject designs have been newly implemented (2003) and no measurable analysis has yet been performed on their effectiveness.

Curtin University’s multidisciplinary feature involves guest lectures by engineering staff reinforcing the importance of mathematics [54].

Focus on mathematical modelling skills in a combined PBL-computer based method further emphasises the need to practise and highlights its closeness to what engineers might expect to experience in the workplace. Referred to as “four leaf clover”, this education model developed by Hadgraft [8] also incorporates “learning from others”

Mathematics for 21st Century Engineering Students: Literature Review

and “learning from literature”, together with “modelling task” and personal skill development”. The involvement of these four major elements is a strong feature, as well as emphasis on practice and repetition.

Subject and curriculum designers and learning support centre proponents have looked at the variability of students [47, 87] and how this may be addressed. Various models and initiatives have been started as a result of this. Streaming students according to their ability is one of them [31, 55-56]. Students’ variability manifests in their knowledge and ability even prior to entering university. In order to address this, universities such as Curtin [54] and Edinburgh [55] have introduced subject groupings that enable students to be streamed according to their academic standing. ETH Zurich [31] has a fast and a slow stream providing students a choice of subjects.

Variability likewise relates to students’ multiple needs and commitments as well as different learning styles. Flexible subjects and support for students with special needs (such as disability or specific health conditions requiring special attention) are useful in this respect. Loughborough University [47] and Central Queensland University [48] are involved in numerous special programs aimed at supporting the broad engineering student population in their learning endeavours. Some of these projects at Loughborough University are the Maths Learning Support Centre [47-48, 87], the HELM project and the Engineering Subject Centre [Refer Appendix B]. These projects are varied and far-reaching. Serving to assist as many students as possible, the programs have given considerable attention to students’ varied needs.

5. Conclusions

An engineering mathematics subject should be a part of a well-designed engineering curriculum for its full benefit to be realised. This has been suggested by the European Society for Engineering Education's (SEFI) proposed hierarchical engineering curriculum [81].

It can also be seen from the literature that if based primarily on learning enhancement, a combination of various effective teaching methods may be the best strategy in teaching mathematics to engineering students since that can capture the value of each method. Some combinations reported to be effective are PBL-computer based [23, 78], multidisciplinary-computer based-part PBL [59] and PBL-multidisciplinary-computer based [61]. It is essential to be able to combine the methods effectively; but in an effective subject design, these methods (computer based, multidisciplinary and PBL) merge well. Most of the reported use of PBL is small group PBL, and with evidence presented for small-group learning's success [13], it is worthwhile noting the benefits of using small groups.

Taking this approach of combining methods to another level, many American universities have integrated theoretical, physical and computer models in a dynamic, projects-based (PBLE) approach. This is demonstrated by mathematics laboratories of University of Delaware and others [41]. The distinguishing feature of this strategy is the presence of physical models. Moreover, having three models which can be viewed in three different perspectives, helps students better recognise the interrelationships between the problem itself, the theory and the physical and computer models. This improves their understanding especially if the integration is accomplished over several semesters, similar to the multi-semester project [75-76, 79] at the University of Massachusetts Lowell. At the University of Delaware, the MEC Lab was used first as a “capstone” course in the fourth year of undergraduate study. In earlier years, there are mathematics subjects that require abstract concept development. We are not aware of any study that investigates how much time is required for individual contemplation of abstract concepts. Britton *et al* [90] attempted to create an index to measure the ability of science students to transfer mathematics, this was later modified by Roberts *et al* [91]. They found that, while students who scored high marks in the in-context questions in the test were able to transfer mathematical knowledge to out-of-context mathematics question, a large proportion of those who scored average marks in the in-context mathematics question were unable to answer the out-of-context questions. This is a worrying finding and indicates that teaching mathematics in context to weaker students limits their ability to use the mathematics in any other context.

Integration may be achieved “laterally” in various degrees. Examples of this are subject designs presented in [23, 59-64]. In this case, there is usually only one subject involved, with different modules utilised in the subject to convey various viewpoints. Integration has also been found to be successfully implemented in an interwoven fashion, that is, both “vertically” and “laterally” at the same time. This is the strategy developed [75-76, 79] at the University of Massachusetts Lowell. Lateral integration takes place across various modules in the same year, while vertical integration is achieved with interconnected materials from most years of the undergraduate program. Interweaving results from simultaneous lateral and vertical integrations, especially when applied to an assortment of modules or projects. Positive effects of

Mathematics for 21st Century Engineering Students: Literature Review

this strategy are evidenced by documented students' direct comments and responses to questionnaires, but not by formal quantitative study.

The multidisciplinary approach in teaching Numerical Methods introduced in Australia and New Zealand [59-64] is relatively new but is very encouraging. Although fundamentally based on the collaboration of different departments in the subject's delivery, it is similar to other strategies reported in that it uses a combination of various teaching methods. The Canterbury model also has additional support [59, 62-64] for students in the form of dedicated tutors, regular drop-in help classes, consultation periods, availability of web-based assistance and help with administrative issues. This may be likened to Math Centres [47-48] of some European universities.

Providing additional support and resources through avenues similar to Loughborough University's Math Centre (Math Centre includes a drop-in centre and a Math Centre website) further helps to enhance student learning [48]. The provision of varied resources also addresses flexibility requirement and responds to students' varied learning styles. This is achieved through the provision of materials and resources ranging from traditional hardcopy handouts and exercises to downloadable video tutorials from the internet, and from personal one-to-one tutorial at the centre to remote 24-hour web-based support.

It would be interesting to investigate more closely the combinations of approaches. A better comparison of their achievements can then be performed. It is also worthwhile considering a comparison of the effectiveness of two or more individual methods particularly in a measurable and controlled manner. No literature reviewed has evidenced such investigations.

It may also be valuable to note that no papers or studies encountered have discussed in detail the impact of cost, time and resources in determining which teaching methods to use. While small-group PBL, computer method or integrated approach may be most effective in enhancing learning, their full implementation may be limited by their associated costs, implementation time and resources.

Although many articles argue the merits of innovative methods of teaching, it is not suggested that the traditional lecture has unconditionally lost its usefulness. Naturally, there needs to be some innovative content in peer reviewed publications so there is not so much material on the most effective styles for conducting traditional lectures and tutorials.

Mathematics for 21st Century Engineering Students: Literature Review

References

- [1] "Year Book Australia 2005," Australian Bureau of Statistics, Jun 2007 [Online] Available: <http://www.abs.gov.au/Ausstats/abs@.nsf/Previousproducts/1301.0Feature%20Article192005?opendocument&tabname=Summary&prodno=1301.0&issue=2005&num=&view=>.
- [2] J. R. Lohmann, H. A. J. Rollins, and J. J. Hoey, "Defining, developing and assessing global competence in engineers," *European Journal of Engineering Education*, vol. 31, pp. 119-131, Mar 2006.
- [3] A. Croft and J. Ward, "A modern and interactive approach to learning engineering mathematics," *British Journal of Educational Technology*, vol. 32, pp. 195-207, 2001.
- [4] P. Kent and R. Noss, "The visibility of models: using technology as a bridge between mathematics and engineering," *International Journal of Mathematical Education in Science and Technology*, vol. 31, pp. 61-69, 2000.
- [5] D. Blockley and N. Woodman, "The changing relationship: civil/structural engineers and maths," Bristol University, in: Maths for Engineering and Science, Learning & Teaching Support Network (LTSN) Maths TEAM Project [Also published in 'The Structural Engineer', 2nd Apr 2002], Edgbaston, Birmingham: Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
- [6] P. Kent and R. Noss, "The Mathematical Components of Engineering Expertise," University of London, End of Award Report: Mar 2002 [Online] Available: <http://www.ioe.ac.uk/rnoss/MCEE/MCEE-End-of-Award-report.pdf>.
- [7] P. Kent and R. Noss, "The mathematical components of engineering expertise: the relationship between doing and understanding mathematics," in *I.E.E. Second Annual Symposium on Engineering Education* London: Institution of Electrical Engineers, Jan 2002 [Online] Available: <http://www.ioe.ac.uk/rnoss/MCEE/Kent-Noss-EE2002-preprint.pdf>.
- [8] R. Hadgraft, "Engineering mathematics for the 21st century," Private Communication (Includes Draft of Paper) Ed, A. Lopez, Ed. University of Melbourne, May 2007.
- [9] P. Kent and R. Noss, "Mathematics in the University Education of Engineers," University of London, A Report to the Ove Arup Foundation: May 2003 [Online] Available: <http://www.ioe.ac.uk/rnoss/REMIT/Kent-Noss-report.pdf>.
- [10] P. Kent and R. Noss, "Should we reassess the role of mathematics in engineering education?" in *LTSN Engineering conference on: Learning and Teaching Support and Resources for Engineers* London: Presentation, Sep 2002 [Online] Available: <http://www.ioe.ac.uk/rnoss/REMIT/LTSN-presentation.pdf>.
- [11] P. B. Henderson, "Mathematical reasoning in software engineering education," *Communications of the ACM*, vol. 46, pp. 45-50, Sep 2003.
- [12] M. J. Pollock, "Introduction of CAA into a mathematics course for technology students to address a change in curriculum requirements," *International Journal of Technology and Design Education*, vol. 12, pp. 249-270, 2002.
- [13] L. Springer, M. E. Stanne, and S. S. Donovan, " f small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis," *Review of Educational Research*, vol. 69, pp. 21-51, Spring 1999.
- [14] ATN, "National project aimed at Engineering Tomorrow's Engineers," in *ATN in Profile: Australian Technology Network of Universities (ATN)*, Vol. 3 Ed.1, Feb 2007 [Online] Available: <http://www.atn.edu.au/newsroom/index.htm>.

Mathematics for 21st Century Engineering Students: Literature Review

- [15] S. Tilli and J. Trevelyan, "Published research on engineering work," *Journal of Professional Issues in Engineering Education and Practice*, To Be Published Oct 2007.
- [16] A. Robson (Reference), "Media Statements 2006: Engineering Skills Package Welcome," The University of Western Australia, Oct 2006 [Online] Available: [http://www.uwa.edu.au/media/statements/media_statements_2006/october/engineering_skills_package_welcome_\(12_october\)](http://www.uwa.edu.au/media/statements/media_statements_2006/october/engineering_skills_package_welcome_(12_october)).
- [17] S. Jordan and D. Yeomans, "Meeting the global challenge? Comparing recent initiatives in school science and technology," *Comparative Education*, vol. 39, pp. 65-81, 2003.
- [18] A. van Langen and H. Dekkers, "Cross-national differences in participating in tertiary science, technology, engineering and mathematics education," *Comparative Education*, vol. 41, pp. 329-350, Aug 2005.
- [19] O. Bringslid, "Mathematical e-learning using interactive mathematics on the web," *European Journal of Engineering Education*, vol. 27, pp. 249-255, 2002.
- [20] K. Hurford, "Forum - An Investment in Australia's Future: why the mathematical science matter," National Strategic Review of Mathematical Sciences Research in Australia, Shine Dome, Canberra: Feb 2007 [Online] Available: http://www.review.ms.unimelb.edu.au/Forum_Hurford.pdf.
- [21] "Active Learning," University of California, Davis, 2007 [Online] Available: <http://trc.ucdavis.edu/TRC/ta/tatips/activelearning.pdf>.
- [22] "Scenes from a Classroom..making active learning work," Centre for Teaching & Learning, University of Minnesota, 2007 [Online] Available: <http://www1.umn.edu/ohr/teachlearn/tutorials/active/what.html>.
- [23] N. Nirmalakhandan, C. Ricketts, J. McShannon, and S. Barrett, "Teaching tools to promote active learning: Case study," *Journal of Professional Issues in Engineering Education and Practice*, vol. 133, pp. 31-37, Jan 2007.
- [24] "Problem-based Learning, especially in the context of large classes," McMaster University, 2007 [Online] Available: <http://www.chemeng.mcmaster.ca/pbl/pbl.htm>.
- [25] "Strategies for the Classroom," Active Learning Online, Abilene Christian University, 2007 [Online] Available: http://www.acu.edu/cte/activelearning/classroom_main.htm.
- [26] S. Sanz, M. F. Iskander, and L. Yu, "Development of an interactive multimedia module on antenna theory and design," *Computer Applications in Engineering Education*, vol. 8, pp. 11-17, 2000.
- [27] R. Taraban, E. E. Anderson, A. DeFinis, A. G. Brown, A. Weigold, and M. P. Sharma, "First steps in understanding engineering students' growth of conceptual and procedural knowledge in an interactive learning context," *Journal of Engineering Education*, vol. 96, pp. 57-67, Jan 2007.
- [28] E. Morgenroth, E. Arvin, and P. Vanrolleghem, "The use of mathematical models in teaching wastewater treatment engineering," *Water Science and Technology*, vol. 45, pp. 229-233, 2002.
- [29] E. Lakoma, "On mathematics teaching for future engineers - in the digital era," *European Journal of Engineering Education*, vol. 27, pp. 279-288, 2002.
- [30] A. Brenner, M. Shacham, and M. B. Cutlip, "Applications of mathematical software packages for modelling and simulations in environmental engineering education," *Environmental Modelling & Software*, vol. 20, pp. 1307-1313, Oct 2005.

Mathematics for 21st Century Engineering Students: Literature Review

- [31] J. Waldvogel, "Teaching mathematics to engineering students at ETH: Coping with the diversity of engineering studies," in *IDEA League Workshop on Mathematics in Engineering* Imperial College, London: Presentation, Apr 2006 [Online] Available: <http://www.math.ethz.ch/~waldvoege/Projects/london.pdf>.
- [32] "Online Learning Materials," School of Mathematics & Statistics, University of South Australia, 2007 [Online] Available: <http://www.unisa.edu.au/math/Webres/online.asp>.
- [33] R. Goering, "Matlab Edges Closer to Electronic Design Automation World," in *EE Times*, Oct 04 [Online] Available: <http://www.eetimes.com/news/design/showArticle.jhtml;jsessionid=WNFN0DJUZXZC2QSN-DLPCKHSCJUNN2JVN?articleID=49400392>.
- [34] M. Zadnik, "Engineering Mathematics 110/120 Technical Outcomes," Private Communication (Includes Technical Outcomes) Ed, P. Broadbridge and A. Lopez, Eds. Melbourne: Received from Curtin University of Technology, Mar 2007.
- [35] M. Zadnik, "Engineering Mathematics 130/140 Technical Outcomes," Private Communication (Includes Technical Outcomes) Ed, P. Broadbridge and A. Lopez, Eds. Melbourne: Received from Curtin University of Technology, Mar 2007.
- [36] P. V. Mtenga and L. K. Spainhour, "Applications of mathematical software packages in structural engineering education and practice," *Journal of Computing in Civil Engineering*, vol. 14, pp. 273-278, Oct 2000.
- [37] G. G. Karady and K. A. Nigim, "Improve learning efficiency by using general purpose mathematics software in power engineering," *IEEE Transactions on Power Systems*, vol. 18, pp. 979-985, Aug 2003.
- [38] A. Naimark, "Applications, MATLAB and linear algebra as a unifying vehicle for the engineering-oriented syllabus," *European Journal of Engineering Education*, vol. 27, pp. 409-424, 2002.
- [39] L. Colgan, "MATLAB in first-year engineering mathematics," *International Journal of Mathematical Education in Science and Technology*, vol. 31, pp. 15-25, 2000.
- [40] C. S. Lee and H. C. Lin, "An interactive framework for teaching power system harmonics," *International Journal of Electrical Engineering Education*, vol. 38, pp. 45-53, Jan 2001.
- [41] "Math512 Contemporary Applications of Mathematics," University of Delaware, 2007 [Online] Available: http://www.math.udel.edu/~rossi/courses/2003_fall/Math512/.
- [42] R. Blair-Editor, "Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College," American Mathematical Association of Two-Year Colleges (AMATYC), Chapter 4: Student Learning and the Learning Environment, Memphis, Tennessee: Nov 2006 [Online] Available: <http://www.beyondcrossroads.com/doc/CH4.html>.
- [43] F. O. Ramirez and C. M. Wotipka, "Slowly but surely? The global expansion of women's participation in science and engineering fields of study, 1972-92," *Sociology of Education*, vol. 74, pp. 231-251, Jul 2001.
- [44] W. Houghton and E. Dunne, "Strategies for teaching mixed-attainment groups in engineering education," *International Journal of Electrical Engineering Education*, vol. 37, pp. 48-66, Jan 2000.
- [45] T. Larkin-Hein and D. D. Budny, "Research on learning style: Applications in the physics and engineering classrooms," *IEEE Transactions on Education*, vol. 44, pp. 276-281, Aug 2001.
- [46] M. S. Townend, "Integrating case studies in engineering mathematics: a response to SARTOR 3," *Teaching in Higher Education*, vol. 6, pp. 203-215, 2001.

Mathematics for 21st Century Engineering Students: Literature Review

- [47] "Mathematics Learning Support Centre," Loughborough University, 2007 [Online] Available: <http://mlsc.lboro.ac.uk/>.
- [48] M. Fuller, "The role of mathematics learning centres in engineering education," *European Journal of Engineering Education*, vol. 27, pp. 241-247, 2002.
- [49] S. Bamforth, A. Crawford, A. Croft, and C. Robinson, "A pre-session course: retaining engineering students through mathematical and transferable skills support," *International Journal of Electrical Engineering Education*, vol. 42, pp. 79-87, Jan 2005.
- [50] "Flexible Learning Centre," Course Information 2007, University of South Australia, 2007 [Online] Available: <http://www.unisanet.unisa.edu.au/courses/course.asp?Course=fkc>.
- [51] J. Dutton, M. Dutton, and J. Perry, "Do online students perform as well as lecture students?," *Journal of Engineering Education*, vol. 90, pp. 131-136, Jan 2001.
- [52] "Teaching with Diversity Checklist," University of Western Australia, 2007 [Online] Available: <http://www.csd.uwa.edu.au/tl/99TDChecklist.htm>.
- [53] "Ensuring Teaching Quality at UWA - Best Practice Pathways," University of Western Australia, 2007 [Online] Available: <http://www.csd.uwa.edu.au/tl/etq/pathways.asp?department=24>.
- [54] M. Zadnik, Private Communication - Engineering Maths Units and Documents ed, P. Broadbridge and A. Lopez, Eds. Melbourne: Received from Curtin University of Technology, Mar 2007.
- [55] J. Christy, "Developing the interface between engineering and mathematics at Edinburgh University," University of Edinburgh (Interview), in: Maths for Engineering and Science, Learning & Teaching Support Network (LTSN) Maths TEAM Project, Edgbaston, Birmingham: Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
- [56] T. Bullough, "Teaching mathematics to first year engineering students with a wide range of mathematical ability," University of Liverpool, in: Maths for Engineering and Science, Learning & Teaching Support Network (LTSN) Maths TEAM Project, Edgbaston, Birmingham: Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
- [57] L. T. S. N. MathsTEAM, "Maths for Engineering and Science," Learning & Teaching Support Network (LTSN) Maths TEAM Project, Edgbaston, Birmingham: pp. 2-3, Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
- [58] R. Haryott, "Foreword," Mathematics in the University Education of Engineers- A Report to The Ove Arup Foundation by P. Kent and R. Noss, London May 2003.
- [59] G. Wake, Private Communication - Mathematics Courses for Engineering Students introduced at the University of Canterbury (Graeme Wake 1998-2003) ed, A. Lopez, Ed. Melbourne: Received from Massey University at Albany, Apr 2007.
- [60] S. Barry, T. Webb, and D. Brooker, "Engineering Computational Methods 1 in support of nomination for Rector's Commendation for Excellent Innovation in Teaching," Private Communication (Includes Report) ed, A. Lopez, Ed. Melbourne: Received from UNSW @ADFA, Apr 2007.
- [61] S. I. Barry and T. Webb, "Multi-disciplinary approach to teaching numerical methods to engineers using Matlab," *Australian & New Zealand Industrial & Applied Mathematics Journal*, vol. 47: Proceedings of the 7th Biennial Engineering Mathematics & Applications

Mathematics for 21st Century Engineering Students: Literature Review

- Conference, Melbourne, Australia (Sep 2005), pp. C216-C230, Aug 2006 [Online] Available: <http://anziamj.austms.org.au/V47EMAC2005/Barry/Barry.pdf>
- [62] G. Wake, "MATH/EMTH171 2006 Course Information (University of Canterbury)," Private Communication (Includes Course Information) ed, A. Lopez, Ed. Melbourne: Received from Massey University at Albany, Apr 2007.
 - [63] G. Wake, "EMTH391-06S2 (U) Engineering Applied Mathematics and Statistics (University of Canterbury)," Private Communication (Includes Course Information) ed, A. Lopez, Ed. Melbourne: Received from Massey University at Albany, Apr 2007.
 - [64] G. Wake, "EMTH 271, MATH 271: Mathematical Modelling and Computation 2 Provisional Syllabus, 2006 (University of Canterbury)," Private Communication (Includes Provisional Syllabus) ed, A. Lopez, Ed. Melbourne: Received from Massey University at Albany, Apr 2007.
 - [65] Y. Umeno, "New ideas on mathematics education in colleges of technology," *Community College Journal of Research and Practice*, vol. 25, pp. 441-444, 2001.
 - [66] P. Kent, "Mathematical components of engineering expertise," *MSOR Connections*, vol. 2, pp. 26-27, May 2002.
 - [67] K. A. Smith, S. D. Sheppard, D. W. Johnson, and R. T. Johnson, "Pedagogies of engagement: classroom-based practices," *Journal of Engineering Education*, vol. 94, pp. 87-101, Jan 2005.
 - [68] T. A. Litzinger, J. C. Wise, and S. H. Lee, "Self-directed learning readiness among engineering undergraduate students," *Journal of Engineering Education*, vol. 94, pp. 215-221, Apr 2005.
 - [69] J. Bransford, "Preparing people for rapidly changing environments (Guest Editorial)," *Journal of Engineering Education*, vol. 96, pp. 1-3, Jan 2007.
 - [70] P. A. Johnson, "Problem-based, cooperative learning in the engineering classroom," *Journal of Professional Issues in Engineering Education and Practice*, vol. 125, pp. 8-11, Jan 1999.
 - [71] J. E. Mills and D. F. Treagust, "Engineering education - is problem-based or project-based learning the answer?," *Australasian Journal of Engineering Education*, vol. Published by the Australasian Association for Engineering Education Inc., 2003, p. [Online] Available: http://www.aeee.com.au/journal/2003/mills_treagust03.pdf.
 - [72] D. Jonassen, J. Strobel, and C. B. Lee, "Everyday problem solving in engineering: lessons for engineering educators," *Journal of Engineering Education*, vol. 95, pp. 139-151, Apr 2006.
 - [73] C. Thomas, "A game show format for first year problem classes in mathematical modelling," University of Birmingham, in: Maths for Engineering and Science, Learning & Teaching Support Network (LTSN) Maths TEAM Project, Edgbaston, Birmingham: Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
 - [74] G. McKay, "New approaches to teaching and learning in engineering at the University of Strathclyde," University of Strathclyde, in: Maths for Engineering and Science, Learning & Teaching Support Network (LTSN) Maths TEAM Project, Edgbaston, Birmingham: Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
 - [75] P. Avitabile, J. McKelliget, and T. VanZandt, "Interweaving numerical processing techniques in multisection projects," in *Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition* Portland, Oregon: American Society for Engineering, Jun 2005 [Online] Available: <http://dynsys.uml.edu/technicalpapers.htm#2005%20-%20ASEE%20Conference>.

Mathematics for 21st Century Engineering Students: Literature Review

- [76] P. Avitabile, T. VanZandt, J. Hodgkins, and N. Wirkkala, "Second order mechanical online acquisition system (R.U.B.E.)," in *Proceedings of the 2006 American Society for Engineering Education Annual Conference & Exposition* Chicago, Illinois: American Society for Engineering, Jun 2006 [Online] Available: <http://dynsys.uml.edu/technicalpapers.htm#2006%20-%20ASEE%A0Conference>.
- [77] PBLE, "A Guide to Learning Engineering Through Projects," Problem Based Learning in Engineering -PBLE (University of Nottingham, Loughborough University, Nottingham Trent University, DeMontfort University): Nov 2003 [Online] Available: <http://www.engsc.ac.uk/downloads/pble/guide2003.pdf>.
- [78] M. J. Duran, S. Gallardo, S. L. Toral, R. Martinez-Torres, and F. J. Barrero, "A learning methodology using Matlab/Simulink for undergraduate electrical engineering courses attending to learner satisfaction outcomes," *International Journal of Technology and Design Education*, vol. 17, pp. 55-73, Jan 2007.
- [79] S. Pennell, P. Avitabile, and J. R. White, "An interdisciplinary, multise semester project relating differential equations and engineering," in *Proceedings of the 2005 American Society for Engineering Education Annual Conference & Exposition* Portland, Oregon: American Society for Engineering, Jun 2005 [Online] Available: <http://dynsys.uml.edu/technicalpapers.htm#2005%20-%20ASEE%20Conference>.
- [80] P. Ernest, "The mathematical attitudes, beliefs and ability of students," University of Exeter, in: Maths for Engineering and Science, Learning & Teaching Support Network (LTSN) Maths TEAM Project, Edgbaston, Birmingham: Mar 2003 [Online] Available: http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf.
- [81] L. Mustoe and D. Lawson (Editors), "Mathematics for the European Engineer A Curriculum for the Twenty-First Century," SEFI Mathematics Working Group, Brussels: Mar 2002 [Online] Available: <https://learn.lboro.ac.uk/mwg/core/latest/sefimarch2002.pdf> (SEFI's Record this Report on: http://www.sefi.be/index.php?page_id=125).
- [82] "Guidelines/Best Practice," Teaching & Learning, The University of Western Australia, 2007 [Online] Available: http://www.teachingandlearning.uwa.edu.au/tl4/teaching_and_learning_committee/resources/guidelinesbest_practice.
- [83] M. Zadnik, "Unit Outlines (Engineering Mathematics 110, 120, 130, 140)," Private Communication (Includes Unit Outlines) ed, P. Broadbridge and A. Lopez, Eds. Melbourne: Received from Curtin University of Technology, Mar 2007.
- [84] "MATH1020 Calculus, Statistics and Probability," UWA Handbooks 2007-Units, University of Western Australia, 2007 [Online] Available: <http://units.handbooks.uwa.edu.au/units/math/math1020>.
- [85] "MATH1010 Calculus and Linear Algebra," UWA Handbooks 2007-Units, University of Western Australia, 2007 [Online] Available: <http://units.handbooks.uwa.edu.au/units/math/math1010>.
- [86] "MATH2040 Engineering Mathematics," UWA Handbooks 2007-Units, University of Western Australia, 2007 [Online] Available: <http://units.handbooks.uwa.edu.au/units/math/math2040>.
- [87] D. Pidcock, J. deSouza, and C. Robinson, "Catalogue of Resources, 6th edition," Mathematics Learning Support Centre, Spring 2006.
- [88] G. Keady, G. Fitz-Gerald, G. Gamble and C. Sangwin. Computer Aided Assessment in mathematical sciences. In *Proceedings of the UniServe Science Conference, Assessment in Science Teaching and Learning*, pp 69-73, September 28, 2006.

Mathematics for 21st Century Engineering Students: Literature Review

- [89] L. Wood (2004). University Mathematics Teaching and Learning. *MERGA Review of Research in Australia 2000-2003*, Flaxton: Post Pressed.
- [90] S. Britton, P. New, M. Sharma and D. Yardley, “A case study of the transfer of mathematical skills by university students”. *International Journal of Mathematical Education in Science and Technology*, Vol. 30, No. 1, pp 1-13, 2005.
- [91] A. Roberts, M. Sharma S. Britton and P. New, “An index to measure the ability of first year science students to transfer mathematics”. *International Journal of Mathematical Education in Science and Technology*, Vol. 38, No. 4, pp 429-448, Jun 2007.

Appendices

Appendix A

Definition of Terms

Mathematics/Statistics Department: an academic unit whose chief responsibility is the teaching of mathematics and/or statistics. This may be a team within a school or a faculty.

Engineering Department: an academic unit whose chief responsibility is the teaching of one or more branches of engineering. This may be a team within a school or a faculty.

Subject: a study of a particular set of topics usually over a period of 12 to 14 weeks which is assessed as an individual element within a degree program.

Engineering Degree Program: the complete 3 to 4 year study program majoring in any strand of engineering.

Engineering Mathematics subject: a subject that is at least 50% mathematical in content taken by students undertaking an undergraduate engineering degree program.

Appendix B

Mathematics Education for Engineering Students in Europe and the United States

The provision of mathematics education to engineering students faces many challenges. These challenges are being addressed by various groups worldwide, including:

- European Society for Engineering Education (SEFI) based in Brussels
[<http://www.sefi.be/>]
- Loughborough University [<http://www.lboro.ac.uk/>] in Leicestershire, UK and other proponents of Mathematics Support Centres (Coventry University and University of Leeds) [<http://mathcentre.ac.uk/>]
- Helping Engineers Learn Mathematics (HELM) consortium
[<http://helm.lboro.ac.uk/> and <http://helm.lboro.ac.uk/pages/consortium.html>]
- School of Mathematics Science & Technology, Institute of Education, University of London have undertaken various projects and studies
[<http://ioewebserver.ioe.ac.uk/ioe/cms/get.asp?cid=54>]
- The Ove Arup Foundation based in the UK, particularly the May 2003 report “Mathematics in the University Education of Engineers”
([<http://www.theovearupfoundation.org/pages/index.cfm>])
- The IDEA League (composed of Imperial College London, Delft University of Technology, ETH Zürich, University of Aachen and Ecole Polytechnique de Paris) in their London Workshop in April 2006 (devoted to Mathematics in Engineering)
[<http://www.idealeague.org/news/newsletter/2006/news29/workshop>]
- The University of Massachusetts Lowell’s interdisciplinary project
[<http://dynsys.uml.edu/>]
- University of Delaware [<http://www.math.udel.edu/MECLAB/>] and eight other American universities supporting MEC Labs (Courant Institute New York, Massachusetts’ Institute of Technology, New Jersey Institute of Technology, Pennsylvania State University, University of North Carolina at Chapel Hill, University of Arizona, North Carolina State University and Georgia Institute of Technology)
- The Connected Curriculum Project (CCP) at Duke University, Montana State University and California Polytechnic State University, San Luis Obispo.
[<http://www.math.duke.edu/education/ccp/aboutccp.html>]

Mathematics for 21st Century Engineering Students: Literature Review

Europe

Europe has in the past decade addressed the broader issue of the teaching and learning of mathematics (for all disciplines, but with particular focus on engineering students). For example, see the description of the LTSN MathsTEAM Project. On the other hand, some engineering organizations (e.g. SEFI) have shown particular interest in mathematics education.

European Society for Engineering Education (SEFI)

SEFI proposed a Core Curriculum based on learning outcomes with a hierarchical structure arranged in four levels [81]. This curriculum assumes a Core Zero prerequisite knowledge that is an essential foundation to move at least to the first level. If this (Core Zero knowledge) has not been achieved upon entry into the program, then Core Zero topics may be taught in support classes running in parallel. Level 1 is Core Knowledge to be covered in Year 1 of the engineering course. This comprises fundamental material essential to every engineering student (with the exception of Computer and Software Engineering students whose curricula follow a different route). Level 2 consists of more advanced knowledge and skills particular to the specific engineering discipline. Practical engineering examples characterise the teaching at this level. Level 3 involves highly specialist knowledge and skills and requires a comprehensive but condensed integration of mathematics and engineering. Generally speaking, the curriculum specifications on mathematics are quite expansive.

Loughborough University

Loughborough University [<http://www.lboro.ac.uk/>] in Leicestershire, UK, has by way of its initiatives and projects, shown considerable commitment to the teaching and learning of mathematics. Its Mathematics Learning Support Centre (housed at the Mathematics Education Centre) [<http://mlsc.lboro.ac.uk/>] holds many different kinds of learning resources and it also provides one-to-one assistance to students studying maths.

Below are some of the main innovative features of Loughborough University's Maths Learning Support Centre [87] :

- HELM Project – delivery of engineering maths through self study open learning
- Engineering student Support Desk – offers web based materials to students and a Power Mac G5 for video and audio processing
- Support available for students with special needs – examples of this type of one-to-one tutoring is given to dyslexic and dyscalculic students
- Videos that show the use of mathematics in modelling engineering problems
- Computers and software – network of 5 PC's; packages perform calculation in algebra, calculus and others, packages also perform graph plotting
- Computer Assisted Learning (CAL)
- Math Centre's dedicated website

Mathematics for 21st Century Engineering Students: Literature Review

- Other resources include leaflets, posters, handouts, workbooks, textbooks, one-to-one tutorial, specialist statistical help, lunchtime refresher courses, study sheets, diagnostic test, graphical calculators and overhead projectors.

Helping Engineers Learn Mathematics (HELM)

The HELM Project was a major curriculum development project undertaken from October 2002 to September 2005 by an alliance of five universities (Loughborough, Manchester, Reading, Hull and Sunderland) [<http://helm.lboro.ac.uk/>]. It was funded by Higher Education Funding Council for England (HEFCE) and was intended to improve the mathematical education of engineering students by the provision of learning resources with emphasis on computer aided learning and assessment. HELM's resources include:

- Fifty workbooks of explanations, worked examples and cases studies
- Web-based Computer Aided Learning (CAL) packages
- Two modes of Computer Aided Assessment (CAA); web-delivered and CD version.

This was a remarkable outcome on a modest budget.

MathCentre

Mathematics Support Centres are housed by a number of universities in the UK. The collaboration between these universities is exemplified by Math Centre [<http://mathcentre.ac.uk/>], a group consisting of universities that run maths support centres (Loughborough University [<http://mec.lboro.ac.uk/>], Coventry University [http://www.mis.coventry.ac.uk/maths_centre/] and University of Leeds [<http://www.maps.leeds.ac.uk/>]) and representatives from the Educational Broadcasting Services Trust and representatives from UK Learning and Teaching Support Networks (LTSN).

MathCentre provides mathematics learning materials free of charge to anyone. The website [<http://mathcentre.ac.uk/>] has learning materials for mathematics learners from different disciplines, including Engineering. The success of MathCentre is evidenced in the high ratings received from the monitored student feedback section of this website.

On the MathCentre website, twelve main engineering mathematics topics are covered including complex numbers, sequences & series, matrices and vectors. Each of these main topics has subtopics that are linked to 'teach-yourself booklets'. These booklets have been rated excellent by users [<http://mathcentre.ac.uk/>]. Aside from 'teach-yourself booklets', the engineering sub-topics also generally contain online exercises and diagnostic tests, as well as video tutorials some of which can be downloaded to an iPod. All of these other learning materials have likewise received high ratings from users. Other disciplines have links to online refresher booklets as well.

Mathematics for 21st Century Engineering Students: Literature Review

The MathTutor [<http://www.mathtutor.ac.uk/index.shtml>] is a set of seven DVD-Rom disks written to assist students of mathematics. MathTutor was created by the same group who created MathCentre and was funded by the HEFCE Fund for the Development of Teaching & Learning and the Gatsby Technical Education Project/Higher Education Academy. The disks contain eighty topics with summary texts and exercises, diagnostics and video tutorials.

Engineering Subject Centre

Engineering Subject Centre [http://www.engsc.ac.uk/us/what_engsc/index.asp] is part of the Higher Education Academy (HEA) and contains two thousand teaching and learning resources. It is based in the Faculty of Engineering at Loughborough University. It was formed in May 2004 from the merger of the Institute of Learning and Teaching in Higher Education (ILTHE), Learning and Teaching Support Network (LTSN) and the Teaching Quality Enhancement Fund National Coordination Team (NCT). The Engineering Subject Centre is the “national centre for all engineering academics in the UK”, it administers support to UK engineering higher education primarily by sharing good practice.

LTSN MathsTEAM Project

The LTSN MathsTEAM Project (funded by the LTSN) carried out an in-depth survey, which led to the publication of three booklets:

- *Maths Support for Students*
[http://www.mathstore.ac.uk/mathsteam/packs/student_support.pdf]
- *Maths for Engineering and Science*
[http://www.engsc.ac.uk/downloads/mathsteam/engineering_science.pdf]
- *Diagnostic Testing for Mathematics*
[<http://www.mathstore.gla.ac.uk/index.php?pid=60>]

Each booklet provides a comprehensive collection of case studies, intended to assist and inspire educators of mathematics. Contributing authors discuss current teaching practice and the barriers and enablers to setting up these learning initiatives. The booklets are intended to assist in the transfer of knowledge within higher education communities informing mathematics educators of effective new and innovative teaching and learning practices. The LTSN scheme has now been superseded by the Higher Education Academy's Subject Centres, including the Maths, Stats and OR Network (<http://www.mathstore.ac.uk/>).

The IDEA League

The IDEA League [<http://www.idealeague.org/about/index>] is an association composed of five leading universities. They are:

- Imperial College London
- Delft University of Technology, Netherlands

Mathematics for 21st Century Engineering Students: Literature Review

- ETH Zurich, Switzerland
- University of Aachen, Germany
- Ecole Polytechnique de Paris

The main objective of IDEA League is to provide a platform for discussing common issues of teaching and curricula and exchange experiences and best practice in those areas. In April 2006, the IDEA League held a workshop ‘Mathematics in Engineering’. This was hosted by Imperial College and included topics such as ‘Mathematics Teaching with Respect to the Diversity of Engineering Subjects’ and ‘Computer Use in Mathematics’.

Jorg Waldvogel [<http://www.math.ethz.ch/~waldvoge/>], in his presentation at the IDEA League ‘Workshop on Mathematics in Engineering’ [31] talked about the various methods in use at ETH Zurich to cope with diversity of engineering students. These include; motivating engineering students, streamlining study, collaboration between mathematics and engineering staff, using practical engineering examples, using software applications in teaching & other innovative ideas such as teaching numerical analysis along with calculus.

Mathematics for 21st Century Engineering Students: Literature Review**The United States**

Mathematics and Engineering Departments at several universities in the United States utilise an integrated form of active learning. Universities examined include the University of Massachusetts Lowell (Interdisciplinary Multi-semester Project on Dynamic Systems) and most universities that support laboratories such as University of Delaware (MEC Lab).

Innovative teaching methods used in these universities are consistent with an integrated approach characterised by active learning environments (noting that problem based learning is one form of active learning). This approach encourages active learning “learning by doing” and combines different models mostly consisting of the physical, mathematical and computer simulation model. Electronic equipment, devices and machines, including computers, are widely used especially in measuring, collecting, storing and analysing data. Projects are often required of students working in small groups [23], challenging them to “rationalise, reconcile, predict and validate” theoretical knowledge against the physical model.

The Modelling, Experiment and Computation Laboratory (MEC Lab)

At the University of Delaware’s Department of Mathematical Sciences, innovative strategies are used to enhance the educational experience of mathematics students. The MEC Lab [<http://www.math.udel.edu/MECLAB>] supports projects where mathematics has direct connection with a physical science phenomenon or experiment, hence its wide scope in engineering. A number of engineering students complete a mathematics minor by taking a 500-level subject with mathematics students in the laboratory.

The MEC Lab is a place where students, teachers and even visitors carry out research and project work related to applied mathematics. It is equipped with PCs running PASCO's Data Studio datalogging software [<http://www.pasco.com/datastudio/>] allowing the use of various sensors and recording information directly to the PC. Data Studio also has some data processing capabilities. In addition to the PC setups, MEC Lab has a range of machine tools, video equipment, power supplies and other miscellaneous materials. Past laboratory activities have been associated with courses such as partial differential equations, mathematical methods, asymptotic methods and fluid dynamics. Examples of engineering concepts handled are magnetic fields, harmonic motion, electrostatic deflections and many others. Research and project work based at the MEC Lab are usually published in the MEC Lab website. Teaching staff are encouraged to incorporate MEC Lab activities in their courses.

Other universities that have similar laboratories in their applied mathematics departments are the following:

- Courant Institute, New York - The AML, Applied Mathematics Lab
- Massachusetts Institute of Technology - Fluids Lab
- New Jersey Institute of Technology - Capstone Lab

Mathematics for 21st Century Engineering Students: Literature Review

- Pennsylvania State University - Fluids Lab
- University of North Carolina at Chapel Hill - Fluid Lab
- University of Arizona - The Applied Math Lab
- North Carolina State University - Instructional and Research Lab
- Georgia Institute of Technology - The ACE Lab

Connected Curriculum Project (CCP)

The Connected Curriculum Project (CCP) is a coordinated effort to develop interactive learning materials for mathematics and mathematics applications, including Engineering Mathematics. The project involves the Mathematics Departments at Duke University, Montana State University and California Polytechnic State University [<http://www.math.montana.edu/frankw/ccp/modeling/topic.htm> - continuous and <http://www.math.duke.edu/education/ccp/aboutccp.html>. These interactive materials combine the Web with specific computer algebra systems such as Mathcad, Mathematica and Maple and contain the following additional features and tools:

- Hypertext Links
- Java Applets
- Sophisticated Graphics
- Realistic Scenarios
- Thought-provoking Questions
- Summary Questions

CCP materials are structured into modules, which are single-topic units [<http://www.math.duke.edu/education/ccp/aboutccp.html>]. These modules require varying amounts of time to complete. This structure makes them flexible to use. They can be used singly or in combination with each other or in combination with other teaching and learning methods. They can also be integrated into courses or used as supplements to usual classroom activities. Lastly, they can be used by individual students and by group of students alike.

Interactive learning materials such as the ones developed by CCP are particularly suited to students of today. The flexibility offered by electronic environment allows designers to incorporate various forms of materials in a way that can convey optimum information, be effective and be interesting to the students. It gives teachers another learning method which can support classroom activities and which places students in an environment in which they are quite comfortable (today's students are comfortable and well-versed with e-learning). The electronic environment allows learning materials to cater to students' varying abilities, needs and situations. It is also a platform for incorporating materials and information that engineering graduates expect to find useful when they join the workforce.

Mathematics for 21st Century Engineering Students: Literature Review

Interdisciplinary, Multi-semester project on Dynamic Systems

Students often do not clearly understand how materials in earlier courses relate to higher level courses. Dr. Peter Avitabile of Mechanical Engineering Department, University of Massachusetts Lowell, and his team attempted to address this by introducing an interdisciplinary, multi-semester project integrating mathematics and engineering [75-76, 79]. This has been applied to a Dynamic Systems course by progressing a series of modules. These modules integrate basic materials from pre-requisite courses, and problem solving materials in later courses. The modules have been set up in pre-requisite courses such as:

- Differential Equations (2nd Year)
- Mechanical Laboratory (3rd Year)
- Numerical Methods (3rd Year)

While topics covered include:

- Numerical Integration/Differentiation
- Visualisation Tools for Understanding 1st and 2nd Order System Response Characteristics
- Understanding Complex Frequency Response Characteristics
- Development of a Virtual Measurement System.

“These modules have been inserted into the pre-requisite courses to enable the students to develop skills on a continuing project that is threaded through the development of the material across several related courses. This culminates in a Dynamic Systems project that forces the students to integrate many previously learned skills in a meaningful manner” [76]

Appendix C

Summary of Reported Quantitative Studies on the Effects of Innovative Teaching Methods

A summary of quantitative studies reported regarding the effects of innovative teaching methods are presented in the following table:

Mathematics for 21st Century Engineering Students: Literature Review

A Summary of Reported Quantitative Studies on Effect of Innovative Teaching Methods

Author/Paper	Number of Respondents/ Length of Time	Main Method Studied	Evaluation Method	Comments/Notes
Nirmalakhandan et al, Jan 2007 (Refereed)	131 in 5 semesters	Integrated Active Learning Approach (Physical, Mathematical & Computer Simulation Models Combined)	Student Evaluations (Surveys), Student Performance	Excellent Affirmation (90% of Respondents), Improved Final Grades (% of Students who passed increased from 70% to 86%)
Taraban et al Jan 2007 (Refereed)	25 (includes science students)	Software Implemented Active Learning (Interactive vs Plain Text both on Online Platform)	Students' verbalisations while completing task (verbal protocol analysis)	Mean verbalisation per screen of 4.30 for Interactive VS approx 2.85 for Plain Text
Duran et al Jan 2007 (Refereed)	Approx 85 (different pre- and post test)	Increased Learner Satisfaction by combination of real examples and virtual simulations (software-based) in a highly interactive PBL context	Pre- and Post- Implementation Questionnaires and Cognitive Tests	Questionnaire – increment positive in 14 of 17 items Cognitive Test – increment positive for all different dimensions
Springer et al Spring 1999 (Highly Cited) (Refereed)	39 reports from 1980 to 1999	Small Group Learning	Standardised mean difference (d-index) effect size	Achievement value 0.51 versus average 0.40; Persistence value 0.46 reduce indolence by 22%; Attitude value 0.55 versus average 0.28
END OF ENTRIES				

Appendix 3: Questionnaire



Mathematics for 21st Century Engineering Students

The Carrick Institute for Learning and Teaching in Higher Education has funded a study to review strategies for the teaching and learning of mathematics and statistics in engineering degree programs throughout Australian universities. The project aims to form a clearer picture of strategies used to teach mathematics and statistics in a technological discipline and how they improve outcomes. Engineering is the focus as a representative field of study.

Completing the Questionnaire

The questions refer to undergraduate teaching of engineering majors and honours students.

Questionnaires may be completed electronically or on hard copy. Email the completed survey as an attachment to slm@amsi.org.au or post to:

AMSI Mathematics for Engineering Project
111 Barry Street
c/o The University of Melbourne
Melbourne
Victoria 3010

Please add extra sheets or use the reverse sides of the pages if more space is required when completing the questionnaire on hard copy.

Definitions

Mathematics/Statistics Department: an academic unit whose chief responsibility is the teaching of mathematics and/or statistics. This may be a team within a school or faculty.

Engineering Department: an academic unit whose chief responsibility is the teaching of one or more branches of engineering. This may be a team within a school or faculty.

Subject: a study of a particular set of topics usually over a period of 12 to 14 weeks which is assessed as an individual element within a degree program

Mathematics Subject: the study of a subject which contains at least 50% mathematics or statistics content usually over a period of 12 to 14 weeks which is assessed as an individual element within a degree program. A Mathematics Subject can be taught by any department.

Engineering Degree Program: the complete 3 to 4 year study program majoring in any type of engineering discipline

Engineering Students: students undertaking an undergraduate engineering degree program

Class: any period of time students participate in timetabled group teaching

Section 1: Subject Information

We have identified the following subject information from your website, please;

1. Verify the information in the tables with any amendments/additions
2. Tick the "50% Maths" column if the subject listed comprises at least 50% mathematics or statistics content
3. Tick the strand(s) of engineer degree programs for with this subject is taken
4. List the typical number of students that attend a single lecture, tutorial and/or laboratory class for each subject listed

1.1 First Year

[illegible]

1.2 Second Year

[illegible]

1.3 Third Year

[illegible]

1.4 Fourth Year

[illegible]

Section 2: Teaching and learning

2.1 What methods are currently used when teaching mathematics and statistics subjects to engineering students? Please indicate the proportion of contact-time students are taught by each method. Please list in the space provided any other teaching and learning methods in use.

Teaching Method	Year 1	Year 2	Year 3	Year 4
Lectures				
Tutorials				
Practical Lab Work				
Maths Lab				
Other				

2.2 What mathematical and statistical software packages do engineering students use in classes? Please specify.

2.3 What challenges has your department faced in the teaching and learning of mathematics and statistics to engineering students in the last 5 years?

2.4 Have you introduced any new methods of teaching and learning (e.g. interactive e-learning, problem based learning, etc)? Please describe these methods and indicate why these new methods were introduced.

Section 3: Characteristics and Needs of Students

3.1 Have there been changes to the academic backgrounds of students entering engineering degree programs, including the effect of changes to high school mathematics and widening university participation?

3.2 Have you noticed any particular changes in the demographics of engineering students in the past 10 years?

3.3 How has your department adapted to these changes?

3.4 Do all engineering students gain the same foundation knowledge of mathematics/statistics? Please explain.

Section 4: Department/Staff Interaction

3.1 Which subjects are taught to engineering students by the mathematics and statistics department?

4.2 Who/which department(s)/committee/center is responsible for overseeing the teaching and learning of mathematics and statistics subjects for engineering students?

4.3 Which parties (engineering staff, mathematics staff, students, Department of Teaching and Learning, industry partners, etc) are involved in the design of mathematics and statistics subjects for engineering students?

Section 5: Assessment

5.1 Are all mathematics and statistics subjects for engineering assessed either wholly or in part by a written exam? If not please list those that are not.

5.2 Which mathematics and statistics subjects for engineers use alternative assessment methods either in place of or in addition to a written exam? Please enter the subject code and the percentage of the final mark each assessment type makes up.

EXAMPLE:

Course Code	Written Exam	Class Work	Group Project	Individual Project	Written Assessment	Online Testing	Other	Other
MA100	70%	10%	20%					

Course Code	Written Exam	Class Work	Group Project	Individual Project	Written Assessment	Online Testing	Other	Other

5.3 How is subject content evaluated? (eg Staff feedback, student feedback, graduate exit survey, Industry feedback). Please explain

Section 6: Industry Relation

6.1 How has your mathematics and statistics curriculum for engineering degree programs changed in the past 5 years in response to changing requirements of employers/perceptions of employment opportunities?

Section 7: Staff Development

7.1 Are there any forums for discussion within the department for the teaching of mathematics and statistics to students of engineering degrees? If so please provide examples.

Section 8: Additional Comments

8.1 Are there any additional comments that your department would like noted in this project?

Appendix 4: National Symposium on Mathematics Education for 21st Century Engineering Students

7th December 2007

Held at RMIT with 16 other Universities connected by a network of access grid rooms (AGRs).

4.1 Symposium Description

The symposium was to energise interest in the teaching of mathematics to engineering students, provide an opportunity for academics to showcase different innovative teaching practice, provide information on acclaimed international teaching and projects for the teaching of mathematics to engineers, create a forum for discussion and to improve and create interdisciplinary relationships between mathematics and engineering educators

4.2 Stakeholders

Engineering academics teaching mathematics to engineering students	Gave several presentations. Many attended as delegates.
Mathematics academics teaching mathematics to engineering students	Gave several presentations. Many attended as delegates.
Council Of Deans Of Engineering	The Carrick funded project Rethinking Engineering Education. The Project Manager for Rethinking Engineering Education (former Dean) gave an invited presentation of the projects findings.
Engineers Australia	Associate Professor Roger Hadgraft was on the organising committee.
Employers of engineers	Invited but unable to participate

4.3 Outcomes

The Symposium allowed for extensive discussion of strategies currently in use or planned for the teaching of mathematics to engineering students and to what extent these methods are improving learning outcomes. A number of different approaches were presented for scrutiny and extensive discussion.

Greater interdisciplinary communication between mathematics and engineering departments was encouraged and promoted. Four engineering professors were involved in discussions – three in attendance, one via AGR.

A better understanding of the necessary university mathematics was developed. The issue of which specific mathematics were necessary was raised after the first presentation and this resurfaced throughout the day with some degree of resolution.

Unanticipated Outcomes

Working closely with the Carrick funded *Addressing the Supply and Quality of Engineering Graduates for the New Century*, sharing information and forging greater links between the mathematics and engineering communities.

At one time, 16 AGR nodes were connected remotely. This was an exemplary launch pad demonstration of the use of this facility in Australia. (See attached "AGR Screenshots")

The presentations by the invited international speakers gave food for thought to a number of delegates. There is potential to link up with the UK HELM project (see below).

4.5 Evaluation

The success of the event was demonstrated by the attendance of in excess of 80 Mathematicians and Engineers from all regions of Australia. Several participants have asked for this symposium to become an annual occurrence.

Written feedback: All participants felt that the international invited talks were Highly effective or effective in meeting the objectives of the symposium, the same feedback was received for the panel discussion. Half the respondents felt the contributed talks were highly effective or effective, some people felt that a couple of the contributed talks were disappointing. All respondents felt that overall the symposium was effective or highly effective in meeting its objectives.

It was felt by respondents that as engineering mathematics courses are similar Australia wide there is scope for uniform change, however, respondents felt that it was a tall order for one symposium and that further symposiums and events are required. We are aware that one university has approached the HELM project in the UK following Martin Harrison's talk and is hoping to introduce the HELM workbooks in the teaching of mathematics to engineers. AMSI is in negotiation with HELM about the possibility of arranging discounted access to their resources for Australian Universities.

Suggestions for dissemination included articles in Math Gazette, a seminar at the ANZIAM conference, web and print, providing some form of resources that would attract people to the website and a publication of the proceedings. We are currently looking into all these courses of action.

Most respondents had not attended a previous Carrick event and said they would attend a similar subsequent event.

Further comments from attendees were very positive, with requests for an annual engineering mathematics symposium and much comment was made about the effectiveness of AGRs for events such as these. Australian speakers implementing more radical teaching and learning styles were requested.

- Presentations, papers and and podcasts available online at: http://www.amsi.org.au/carrick_seminar_program.php
- Some snapshots of the seminar in progress at multiple centres

